Problem Chosen	2022 HiMCM Summary Sheet	Team Control Number
A		12519

Summary

Problem A was composed of 4 sub-tasks: modeling the population of a honeybee colony over time, conducting an analysis of which factors most impacted the population, estimating the optimal number of beehives required to pollinate a 20 acre plot of land, and creating an infographic to display our findings to a general audience.

To model the population of a bee colony, we identified four major areas that impacted bee populations: the birth rate, the death rate caused by overworking, the death rate caused by predators, and the death rate caused by disease. To best account for each individual factor, we developed a rate equation for each sub-factor. We identified three criteria that played a significant role in determining the birth rate: the egg-laying rate, changes in the season, and the carrying capacity of a colony. From our background research, we established that queen bees laid a different amount of eggs in each season, ranging from 0-3000 eggs daily, and bee colonies were able to maintain a maximum of 80,000 bees.

When considering internally influenced deaths, the bee population was split into two subgroups, forager bees and non-forager bees, in order to represent deaths related to overworking. In addition, deaths attributed to predators were also analyzed. The Volterra-Lotka Predator Prey system of differential equations was used to model a cyclic pattern for these externally influenced deaths. The final sub-equation, death due to disease was modeled by a seasonally weighted exponential growth model using the rate of disease spread.

These subcategories were compiled into a singular differential model that combined all four sub-equations to best establish the population of the colony over time.

Our approach to analyzing which factor most impacted our differential population model uses radar maps to classify parameters of high impact and a testing strategy that predicts populations with varied parameters ($\pm 20\%$) using a Python implementation of Euler's Method. These parameters included the rates used to establish our model as well as additional parameters, such as initial population, fertilization rates, and maximum daily offspring.

To estimate the number of beehives needed to pollinate a 20-acre plot of land, we analyzed the average number of forager bees in a colony, the distance covered by a single hive, and the number of flowers in the given area of land. We utilized the Gaussian distribution curve and geometric principles to decide that only one beehive is needed to pollinate a 20-acre piece of land. Furthermore, we adapted our model to analyze larger areas of interest and determine the optimal number of beehives to pollinate the area.

Finally, we created a non-technical infographic to communicate our findings to a general audience.

Contents

1 Introduction

1.1 Background

Although we cannot answer the question "to be(e), or not to be(e)," by Shakespeare, we *can* develop a mathematical model to determine the population of a honeybee colony over time. Honeybees play a vital role on Earth by pollinating trees and plants in our ecosystem. However, beekeepers across the globe have noted major declines in colony stability, attributed to external stress factors such as disease, predators, and environmental changes, as well as internal stress factors such as overworking. To evaluate the decline of honeybee populations worldwide from 2007, we developed a model that takes in numerous characteristics, such as birth rates, death rates, predator versus prey rates, and disease rates.

1.1.1 Bee Typology

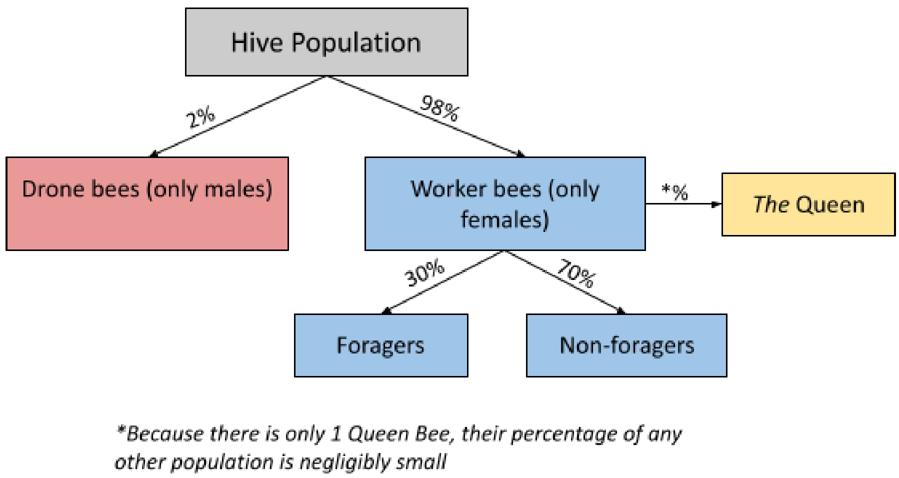


Figure 1: Honeybee social hierarchy [1] [2].

- Drone Bees: necessary for reproduction and up-keeping the population.
- Worker Bees: protect the hive as well as the queen.
  - Forager Bees: go out into the world to pollinate the flowers, trees, and crops, while bringing home nectar and pollen to feed the hive.
  - Non-Forager Bees: remain within the hive, creating cells for new baby bees, feeding the larvae, and ensuring that the hive is well-maintained.
- Queen Bee: lay and fertilize eggs.

Overall, honeybees serve a major role in the agricultural communities, as they pollinate a vast majority of our crops. Despite their significance, from 2007 onwards, the honeybee population has experienced a significant decline. This change in population is partially attributed to a phenomenon known as Colony Collapse Disorder (CCD), in which a majority of the worker bees in a hive disappear as a result of overworking and other external factors. Interestingly, these bees leave behind a colony with a healthy queen and a plethora of food and supplies. Collapsing colonies often give little to no indication of the imminent collapse and appear to be perfectly healthy since worker bees tend to die very quickly into the foraging season [3].

1.2 Problem Restatement

Question 1: Create a model that represents the population of bees in a colony over time.

Question 2: Determine what factors (e.g., lifespans, egg-laying rates, fertilized/unfertilized egg ratios, etc.) have the greatest impact on the honeybee population.

Question 3: Estimate how many bee hives are necessary in order to pollinate a 20-acre piece of land.

Question 4: Create a one-page blog or infographic that conveys our results in a non-mathematical, so a general audience can understand our findings.

1.3 Problem Analysis

Question 1: There are two ways that bee populations (in general) are affected: births and deaths. Many factors affect birth rates, including the number of eggs laid, the fertilized vs. unfertilized egg ratio, seasonal effects/food availability, and population threshold. Additionally, we identified three causes of death: (a) death by work or natural death, (b) death by disease, and (c) death by predators. These three causes of death, in addition to the birth rate, mean that we have 4 rates to keep track of.

Question 2: The factors we decided to test our model on include rates and parameters. The rates that could have an impact on bee population includes those that were used in order to create our population model. This includes birth rate, death rate, predator vs. prey, and disease rate. Additionally, the parameters that could impact the bee population include initial population, fertilization rates, and maximum daily offspring. Using these rates and parameters, we can conduct a sensitivity analysis on our model.

Question 3: In order to analyze the number of hives needed to pollinate a given amount of land, we need to determine the number of bees that will travel specific distances, the number of flowers in the given amount of land, and the number of flowers that will be pollinated based on the number of bees there are.

1.4 Assumptions

1.4.1 Assumptions for Parts 1 & 2: Cyclical Population Model & Analysis

- **Assumption 1:** The average number of bees per hive is 40,000.
  - This value is in the range that the problem specified and is the average of the maximum and minimum values.
- **Assumption 2:** Bees have predators.
  - This is an important factor to consider when calculating the population of bees over time.
- **Assumption 3:** Queen bees live for 4 years.
  - Queens live for longer than worker bees as they are essential for reproduction. A queen bee lives for an average of 4 years [4].
- **Assumption 4:** A colony will collapse if it has less than 4,000 workers.

  - A beehive is not sustainable unless there is a certain amount of workers maintaining the hive. Without at least 4,000 workers, a beehive will collapse [5].

- **Assumption 5:** The carrying capacity of a hive is 80,000 bees and anything over is negligible.
  - As stated in the problem, beehives contain about 20,000 to 80,000 bees. Once a colony reaches 80,000 bees, the hive cannot support these bees so they either move to new colonies or die.

- **Assumption 6:** Online statistics used in our hierarchy tree are representative of the local hive populations.
  - The statistics used in the hierarchy tree were researched as an overall average; therefore, we are applying them to our model in order to identify the percentage of each type of bee [3].

- **Assumption 7:** All worker bees that are not forager bees are non-forager bees.
  - Worker bees are divided into two groups: foragers and non-foragers. All worker bees fall into one of the two categories.

- **Assumption 8:** Queen bees lay the most eggs in July and August.
  - The summer season is the peak foraging season; therefore, more eggs are laid in the summer to account for the increase in loss of bees during that time. [6]

- **Assumption 9:** The average lifespan of bees in the winter is higher compared to the summer.
  - Since the foraging season is in the summer, bees are working harder to maintain the hive as forager bees begin to forage. This causes the lifespan of bees to be shorter during the summer because they are working harder than they do in the winter. [7]

- **Assumption 10:** If a bee is kicked out of the hive, specifically drone bees in winter, they are presumed to be dead and included in our death count.
  - Drone bees are kicked out of the hive during the winter as that makes it easier to maintain the hive since drone bees are only essential for reproduction. It is assumed that when drone bees are kicked out of their hives, they do not survive. [2]

1.4.2 Assumptions for Part 3: (N)estimation

- **Assumption 11:** A 20-acre piece of land will have dimensions that have a ratio of 4:5.
  - We assumed that the land would be a rectangle because of its use in crop production.

- **Assumption 12:** There are 10,000 plants per acre of land.
  - After conducting research on the average number of different plants that can fit in an acre of land that also benefit from pollination, we estimated the average number of plants in general that would fit into an acre of land.

- **Assumption 13:** There are 3 flowers per plant.
  - Since plants can contain multiple flower and different plants contain varying numbers of flowers, the number of flowers on a plant must be defined when calculating the number of flowers in the area. In order to encompass that different plants have different amounts of flowers, we assumed that the average flower to plant ratio is 3.
- **Assumption 14:** 90% of bees travel only 6 km away from their hive and 10% of bees forage in between 6 km and 20 km away from their hive
  - Bees typically stay within 6 km of their hive; however, they will travel up to 20 km away from their hive. We assumed that the majority of bees, 90%, stay within 6 km of their hive while the remaining 10% venture past 6 km up to 20 km away from the hive.

2 Part 1. Cyclic Population Model

2.1 Brief Overview

In order to evaluate the population of bees in a colony over time, we considered birth, natural death, predator vs. prey, and pathogenic rates. When analyzing bee colonies, these four factors contributed the most to the change in populations. By summing these rates and weighting them based on their impact to the overall population growth rate, the final population model by integrating the rate can be accurately determined. Using this model, we are able to visualize the effects of certain factors on the bee population to identify what the biggest threats are to bee colony populations. The variables and parameters that were measured and used to develop the four sub-models are discussed below.

Variable	**Symbol**	**Description**
Maximum Daily Offspring	L	According to [1], the maximum amount of daily eggs a queen bee lays is 3000 eggs.
Egg Fertility Rates	R	98% of all eggs the queen lays are fertilized, resulting in a female bee [2].
Total Population of Bees	P	The total population of bees ranges between 20,000 and 80,000 when a colony is alive and stable.
Population of Forager Bees	P_f	The population of forager bees is 30% of the total population [1].
Population of Non-Forager Bees	P_{nf}	The population of non-forager bees is 70% of the total population.
Years	T	The number of years T is the number of years after 2007.
Population of Predators	Y	-
Deaths	D	The number of deaths from work-related fatalities or life-expectancy deaths.

2.2 Determining Rates That Affect the Population

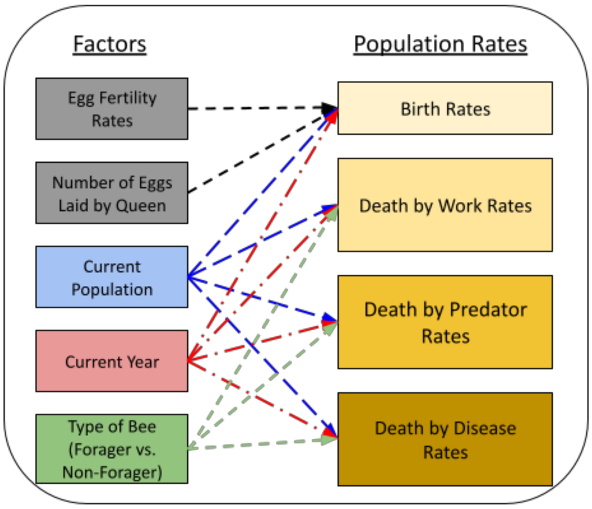


Figure 2: Parameters of each rate.

2.2.1 Rate #1: The Birth Rate

Our first step in finding the birthrate of the population is determining the number of offspring produced per day, which is found from the number of eggs produced per day. However, given that not all eggs successfully become bees, we also need to apply the fertilization rate to the number of births to find the number of offspring produced each day. In addition, the assumption was made that the maximum amount of daily eggs a queen bee lays in any given day is 3000 eggs, and the fertilization rate is 0.98.

A basic birth rate function is defined below, where L is the maximum number of eggs laid each day and R is the ratio of eggs that are fertilized and will create offspring. The function for the number of births in a given year is defined as $B(t)$, so the differential equation $\frac{dB}{dt}$ represents the birth rate function. Note that although $B(t)$ is a function of time, our initial equation is constant.

$$\frac{dB}{dt} = 365 \cdot L \cdot R$$

However, this basic model does not account for days where the queen produces less eggs than the maximum potential. Such an event occurs in months with decreased pollen, like the winter months, so a continuous function scaled between 0 and 1 was multiplied into the expression shown below. The period of the sine function was shrunk to 1 and transformed upwards by 1 to represent that every year would experience the same distribution of eggs laid since the egg laying rate of a queen bee remains the same.

$$\frac{dB}{dt} = 365 \cdot L \cdot R \cdot \frac{(\sin(2\pi t) + 1)}{2}$$

When assuming the maximum egg laying rate to be 3,000 eggs per day and 98% of eggs are fertilized, this equation can be simplified to the below expression.

$$\frac{dB}{dt} = 3000 \cdot 0.98 \cdot 365 \cdot \frac{(\sin(2\pi t) + 1)}{2} = 547500 \sin(2\pi t) + 547500$$

Lastly, a function of time was used to model the decrease in birth rates when it approaches the colony's carrying capacity of $80,000$ bees. Since fewer bees will be produced as the population approaches 80,000, we multiply the previous expression by a factor of $\frac{80000-P}{80000}$ to simulate logistical growth, obtaining the final result:

$$\frac{dB}{dt} = \frac{80000 - P}{80000} \cdot 547500 \sin(2\pi t) + 547500 \quad (1)$$

2.2.2 Rate #2: The Natural Death Rate

The natural death rate (caused by overwork) must be split into two parts, the death rate for forager bees and for non-forager bees. This is because the death rates for each section differ. The forager bees, who venture outside of the safety of the hive, encounter different hazardous scenarios compared to non-forager bees, resulting in varied death rates.

We defined a function $D(t)$ which measures the number of work-related fatalities for a given year. Thus, the rate of deaths could be measured by $\frac{dD}{dt}$, assuming that $D(t)$ is differentiable. The rate function of deaths $\frac{dD}{dt}$ would be measured in the number of deaths per year. When evaluating the death rate in two parts—forager bees and non-forager bees—we defined two rates: $\frac{dD_f}{dt}$ for forager bees and $\frac{dD_{nf}}{dt}$ for non-forager bees so that

$$D(t) = D_f(t) + D_{nf}(t)$$

and

$$\frac{dD}{dt} = \frac{dD_f}{dt} + \frac{dD_{nf}}{dt}$$

Forager bee populations have a daily fatality rate of 30%, so we can represent the total percent of forager bees who die with the expression: $(1 - 0.7^{365t})$ [1]. Furthermore, the forager bee population accounts for 30% of the total bee population, which can be modeled by $P_f = 0.3 * P$. Combining these 2 fragments, the final death rate for foraging bees can be modeled as

$$\frac{dD_f}{dt} = (1 - 0.7^{365t}) \cdot 0.3 \cdot P$$

The rate function $\frac{dD_f}{dt}$ represents the death rate per year. However, this function does not account for the non-constant death rates in the different seasons—winter, spring, summer, and fall—where winter has the lowest rates of death and the summer has the highest rates of death. To remedy for this, the rate was multiplied by a sinusoidal wave function with a period of 1 and a vertical transformation of 1 upwards to prevent negative death rates. Thus, the death rate would become

$$\frac{dD_f}{dt} = (1 - 0.7^{365t}) \cdot 0.3P \cdot \frac{(\sin(2\pi t) + 1)}{2} \quad (2)$$

The non-forager bee population faces a daily fatality rate of around 40% [4]. To model this death rate, the equation shown below encompasses the percentage of fatalities, the seasonal changes of death rates, and the current population size to determine the probability of mortality for non-forager bees. The number of non-forager bee deaths is greatest in the winter and least in the summer [7]. To account for this discrepancy, we use a transformed cosine function. Additionally, the population of non-forager bees is seventy percent of the total population.

$$\frac{dD_{nf}}{dt} = (1 - 0.6^{365t}) \cdot 0.7P \cdot \frac{(\cos(2\pi t) + 2)}{2} \tag{3}$$

The total death rate function is a linear combination of the forager death rate and the non-forager death rate as shown below, measured in the number of deaths per year.

$$\frac{dD}{dt} = (1 - 0.7^{365t}) \cdot 0.3P \cdot \frac{(\sin(2\pi t) + 1)}{2} + (1 - 0.6^{365t}) \cdot 0.7P \cdot \frac{(\cos(2\pi t) + 2)}{2} \tag{4}$$

2.2.3 Rate #3: The Death Rate Caused by Predators

We used the Volterra-Lotka model (a system of differential equations) to represent the death rate caused by predators. The Volterra-Lotka model is commonly used for analyzing the population dynamics of competing species. The predator-prey model was only applied to the forager bee population because forager bees are the only bees to leave the hive. Hives are rarely attacked by predators and an attack on the hive may cause severe damage or entire collapse of a bee colony population, making it irrelevant to our model scenarios.

Let the population of the forager bees be defined as $P_f(t)$ and the population of all natural bee predators be defined as $Y(t)$. Thus, the rates of change of each population is defined below, where A, B, C, and D are constants.

$$\frac{dP_f}{dt} = AP_f - BP_fY \qquad \frac{dY}{dt} = -CY + DP_fY \tag{5}$$

We used additional Volterra-Lotka models to determine the ratio between the coefficients, optimizing them so A, B, C, D equal to 50, 6, 1, and 0.5 respectively. A represents the raw birth rate of bees. In this context, "raw" means without any external factors (e.g., predators, disease). B represents the raw loss rate of bees because of predators, C represents the raw fatality rate of predators, and D represents the raw birth rate of predators.

A was chosen to be 50 because the bee population would grow much faster than the predator population if there were no external dangers or harm. B was chosen to be 6 because the bee population loses some but not many members to predators. This is a significantly lower number than 50 because the correlation between predator and bee populations is not as significant. C was chosen to be 1 because bee predators do not suffer as many losses compared to bees. Lastly, D was determined to be 0.5 because the bees reproduce more often than bee predators.

This system of differential equations yields the following solution:

$$DP_f - C\ln(P_f) + BY - A\ln(Y) = C_1$$
$$0.5P_f - \ln(P_f) + 6Y - 50\ln(Y) = C_1$$

When the values of C_1 are varied, the predator-prey relationship varies around a central critical point of stability. This means that there is no asymptotic or unstable behavior within a bee-predator system. The orbital map for this system is shown below, modeling the possible predator and bee populations.

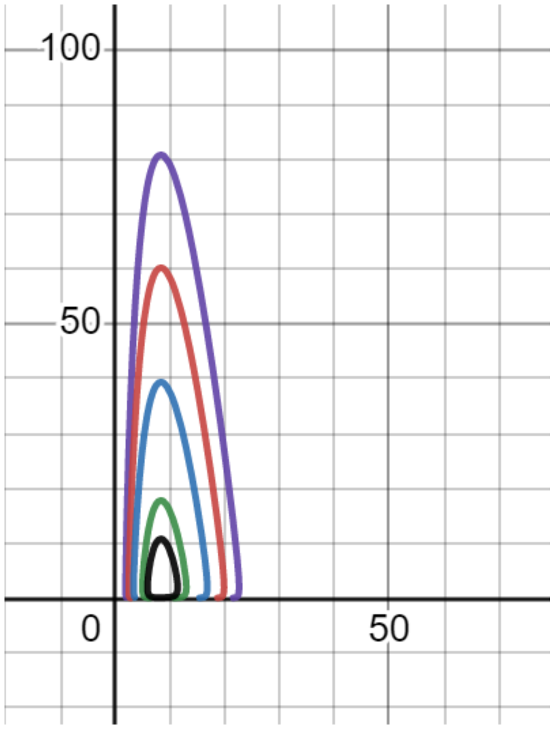


Figure 3: A orbital map modeling the predator and bee populations in tens of thousands.

To solve for $\frac{dP_f}{dt}$ to find the rate of change of population, we solved for Y in the above equation with $C_1 = -30$. The value -30 was chosen to be the optimal integration constant for modeling this bee population scenario because the orbital graph shown above relating predator and prey populations represents the range from twenty to eighty thousand bees.

$$Y = 8.3 \cdot \log(0.17(\exp(\frac{0.00000735P}{\sqrt{0.294P}}))^{0.04})$$

This function for Y was then plugged back into the first equation of Equation 5 for $\frac{dP_f}{dt}$ to model the rate of change for the forager population in bees per year. Since $P_f = 0.3P$, we multiplied the new expression by 0.3, yielding the equation below.

$$\frac{dP}{dt} = 14.7P + 1.764P \cdot 8.3 \cdot \log(0.17(\exp(\frac{0.00000735P}{\sqrt{0.294P}}))^{0.04}) \tag{6}$$

2.2.4 Rate #4: The Rate of Deaths Caused by Disease (or Pathogens)

In order to model the probability of a bee being attacked by a pathogen or encountering a disease, we modified a basic exponential function with a damping trigonometric function. Since the probability of a forager bee contracting a virus or disease is higher than that of a non-forager bee, the rate at which each type of bee becomes infected and dies is defined separately. The function $V(t)$ is defined as the number of bees who died from pathogenic causes, and the derivative $\frac{dV}{dt}$

represents the rate of change of the number of bees who died. Furthermore, the rate at which the bee population dies from disease can be separated into foraging and non-foraging bees so that

$$V(t) = V_f(t) + V_{nf}(t)$$

and

$$\frac{dV}{dt} = \frac{dV_f}{dt} + \frac{dV_{nf}}{dt}$$

First, the rate at which a forager bee ($\frac{dV_f}{dt}$) becomes infected and dies was modeled using an exponential equation where the rate of growth is directly proportional to the current population of forager bees (P_f). We defined the proportionality constant k_1 for the equation below as 0.03 based on similar ratios from previous disease spread models. This constant proves optimal because it will allow the model to modify the rate at which bees become infected.

The exponential growth model was multiplied by a trigonometric damping factor to account for the seasonal changes in disease spread. In winter months, the cosine of the time would be near or at 0 to show a lower rate of disease spread. In summer months, the cosine of the time would be near or at 1 to show the higher rate of disease spread, which aligns with observations on bee populations. This equation is modeled as shown below.

$$\frac{dV_f}{dt} = k_1 P_f(\cos(2\pi t) + 1)$$

In the populations containing non-foraging bees, the rate of disease spread is lower because worker bees in the hives are not exposed to as many pathogens compared to those outside of the hive. This would justify a new proportionality constant k_2 as 0.01 for an exponential growth model in non-foraging bees disease fatalities. This lower proportionality constant indicates that the rate of growth is lower and the disease does not easily spread through the non-foraging bee population. The equation modeling non-foraging bees with population (P_{nf}) is shown below.

$$\frac{dV_{nf}}{dt} = k_2 P_{nf}(\cos(2\pi t) + 1)$$

The total rate of change of the population based on pathogenic causes is the sum of the rates for foraging bees and non-foraging bees, as shown below. To express this in terms of the total population, the assumption is carried on from previous sections that the population of foraging bees is 30% of the total population and the population of non-foraging bees is 70% of the total population.

$$\frac{dV}{dt} = 0.03 \cdot 0.3P(\cos(2\pi t) + 1) + 0.01 \cdot 0.7P(\cos(2\pi t) + 1) \tag{7}$$

2.3 Combining the Four Rates

From the four rates above—birth, death by work, death by disease, and death by predators—we constructed a singular model that encompasses all of these rates to predict population growth and values. We assigned a negative coefficient to the rates related to death and a positive coefficient to the birth rate. The differential population model that contains the four rates from Equation 1, 4, 6, and 7 is shown below.

$$\frac{dP}{dt} = \frac{dB}{dt} - \frac{dD}{dt} + \frac{dP}{dt} - \frac{dV}{dt} \tag{8}$$

$$\begin{aligned} \frac{dP}{dt} = & \left[\frac{80000 - P}{80000} \cdot 547500\sin(2\pi t) + 547500\right] \\ & - \left[(1 - 0.7^{365t}) \cdot 0.3P \cdot \frac{(\sin(2\pi t) + 1)}{2} + (1 - 0.6^{365t}) \cdot 0.7P \cdot \frac{(\cos(2\pi t) + 2)}{2}\right] \\ & + \left[14.7P + 1.764P \cdot 8.3 \cdot \log(0.17(\exp(\frac{0.00000735P}{\sqrt{0.294P}}))^{0.04})\right] \\ & - \left[\frac{dV}{dt} = 0.03 \cdot 0.3P(\cos(2\pi t) + 1) + 0.01 \cdot 0.7P(\cos(2\pi t) + 1)\right] \end{aligned} \tag{9}$$

2.4 Estimating the Combined Population Model

We already know that the male and female populations change throughout the seasons due to each of their select roles and responsibilities. The female worker bees establish new cells for the queen to lay eggs in, the female forager bees nourish the colony by bringing back pollen and nectar, and the male drones will mate with the queen to bring forth the next generation of the colony.

As the seasons change, the rate of pollen and nectar production increases, leading to a higher demand for forager bees. Without an increase in forager bee numbers, the current forager bees are overworked and die faster, leading to CCD. As the need for forager bees increases, the worker bee population must also grow in order to build the cells for the new baby bees that will become foragers. In addition to the worker bee population, the colony requires more drone bees to reproduce with the queen to create the eggs itself.

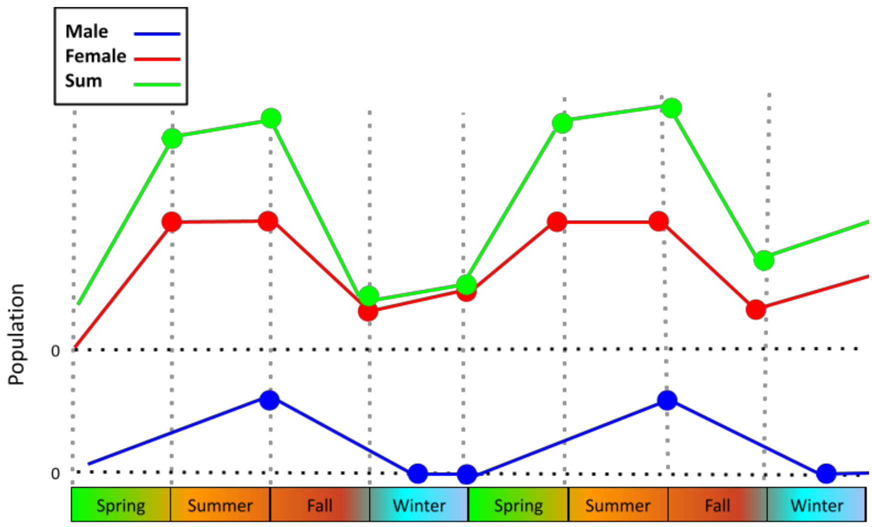


Figure 4: An estimation model for the total population of bees.

After we determined that there will be a periodic difference in population compared to the 4 seasons, we analyzed and developed Figure 4. In the spring, as flowers begin to bloom, pollen and nectar production starts back up, leading to a rise in both female and male bee populations. When summer hits, pollen and nectar amounts are at their highest, creating stability in the female population as the male population increases yet again to account for the high volume of baby bees.

As fall approaches and flowers begin to die off, the demand for forager bees also dies off, leading to a steep decline in the male population and a gradual decline of the female population. As winter hits, the drone bees are kicked out of the hive since there is little to no need for additional children as the colony has limited amounts of food and can not maintain a large population. The female population stabilizes once more during the winter, albeit at a lower population and remains fairly stable until spring returns again, repeating this cycle.

3 Part 2. Cyclic Population Model Analysis

3.1 Combined Model Analysis

Although the differential equation involving $P'(t)$ is not integrable, an implementation of Euler's method to approximate particular solutions is possible. To achieve this, we developed a Python script (Appendix 2) based on the following recursive sequence given an initial time and population.

For the initial analysis of the model, the hive was set to 20,000 bees on the year 2007. In addition, the step size for was set to 0.1 to accurately determine the approximate solutions throughout the year's months.

$$t_0 = 0$$

$$P_0 = 0$$

$$\Delta t = 0.1$$

$$t_{n+1} = t_n + \Delta t$$

$$P_{n+1} = P_n + P'(t_n) \cdot \Delta t$$

Our code simulates finding the population growth at each point in time and predicts a new population for the next time. The model shows periodic trends when tested with an initial value of 20,000 bees. The periodic trend would continue to repeat without end. This model also oscillates between the 35,000 and 70,000 bees when initialized with a starting value of 20,000 bees. The length of each period is exactly one year, which shows that the bee population maintains a stable path within the range of the acceptable colony sizes. Figure 5 shows the graph of the first two years with a period of exactly one year. Figure 6 shows the graph for 10 years after 2007.

Our differential model is justified in terms of seasonal trends since Figures 5 and 6 closely resemble the estimation curve from Figure 4. The seasonal changes match because winter shows smallest bee populations and summer is correlated with larger bee populations.

3.2 Sensitivity Analysis

Sensitivity analysis was conducted by varying the parameters within a 20% range. The parameters of the maximum daily egg laying rate and the fertilization rate were analyzed. Since

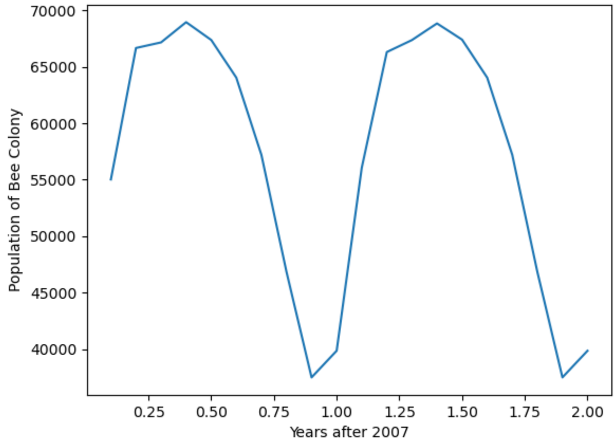


Figure 5: Euler's approximation to the combined model differential equation.

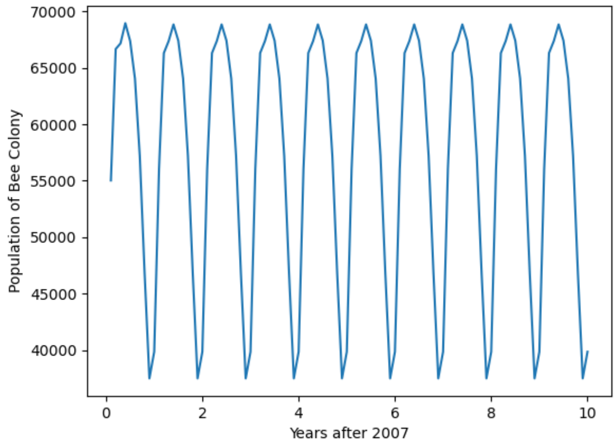


Figure 6: Euler's approximation to the combined model differential equation continued.

we defined the default value of the maximum daily egg rate and the fertilization rate to be 3000 and 0.98 respectively, the values for the egg rate from 0 to 3000 were analyzed as well as fertilization rates from 0 to 1.

These new parameters were passed into the Euler's method particular solution approximation process which affect the birth rate and death rates. The varying population results were recorded at the time t = 10 years to determine the effect of these parameters on the population. It was found that the egg laying rate has a greater impact of population based on percentage change. For example, a 25% growth in eggs laid per day from 1,500 to 2,250 would increase the population in ten years by a 50% from 13,000 to 21,000 bees as shown in Figure 7.

We also developed radar charts comparing the effect of each rate on the population when analyzing the individual rates (birth, death, predator-prey, disease) in each interval of time. The radar charts also indicated that the birth rate and daily egg rate impact the population growth most significantly.

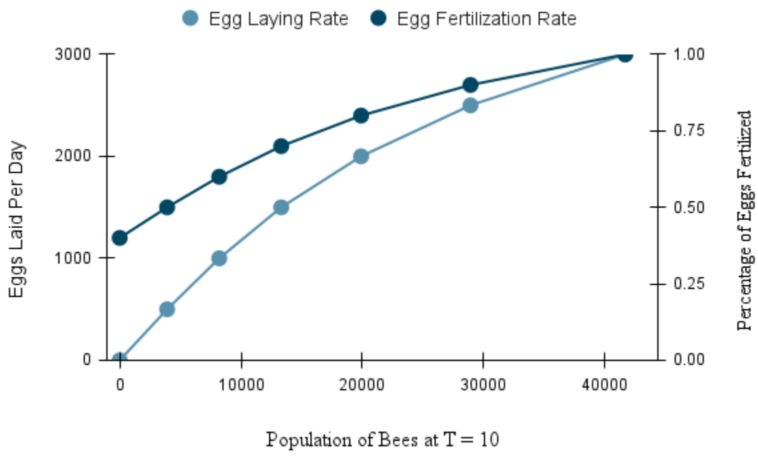


Figure 7: Effect of parameters on bee population.

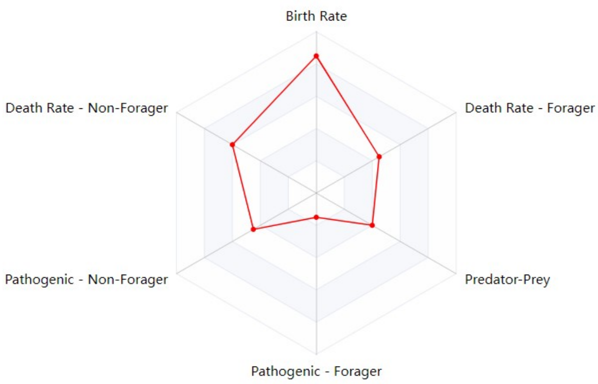


Figure 8: Radar chart of the five components of population rates.

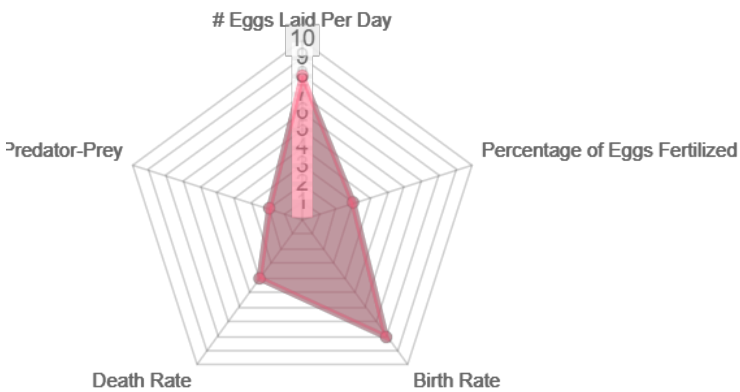


Figure 9: Radar chart of the parameters and affected rates.

4 Part 3. (N)estimation

4.1 Brief Overview

In this problem, we are given that there is a 20-acre plot of land and a model needs to be created to show how many bee hives are needed in order to keep this plot of land pollinated, considering the land is filled with a plant that benefits from pollination. In order to solve this problem, the factors we decided to consider is making sure all flowers on the plot of land are being reached by the bees as well as that there are enough bees to pollinate all the flowers on the piece of land.

4.2 20-Acre Area

Given that the plot of land is 20 acres, we assumed that the ratio of the sides of the plot is 4:5. Using this information, we calculated the dimensions of the plot in m^2 given that 20 acres is equal to 81,000 m^2. It was found that the dimensions would be 318.198m by 254.56m.

4.3 Beehive Radii

Using the Gaussian distribution curve (also known as the normal curve), we were able to create a model that calculates the distribution of forager bees relative to their traveling distance from the hive. Assuming that the average number of bees in a hive is about 40,000 and only 29.4 percent of all bees travel outside the hive, there are, on average, 11,760 forager bees in a colony. We then analyzed the percentage of bees that will travel only 6 km away from their hive compared to the percentage of bees that will travel further than 6 km away from their hive.

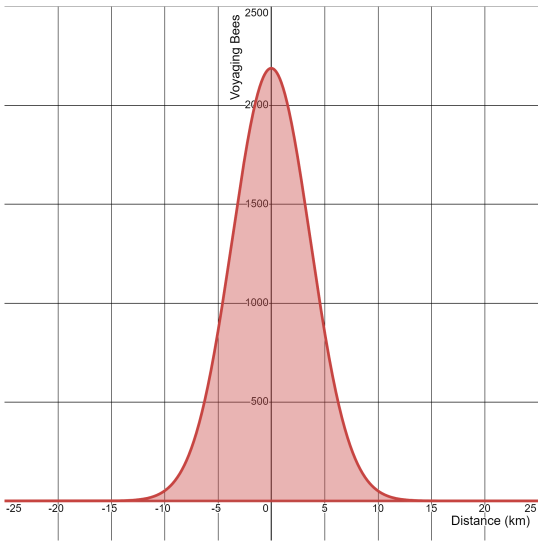


Figure 10: Gaussian distribution curve (or normal curve) of voyaging distances

Assuming that 90% of bees would stay within 6,000 m of their hive while the remaining 10% would fly in between 6,000 m and 20,000 m of the hive, a normal distribution curve was constructed (Figure 10). By applying the data to a normal model, the standard deviation can be identified as 3,647 m with the mean representing the beehive at 0. Using this information, we were able to create a model that shows how many hives are necessary on a 20-acre piece of land as well as where they should be located on the land.

Additionally, we represented a beehive as a point with two circles around it, which can be seen in Figure 11. The red circle represents the 6,000 m radius around the beehive where 90% of bees fly around while the green dashed circle represents the 20,000 m radius around the beehive where 10% of bees fly around.

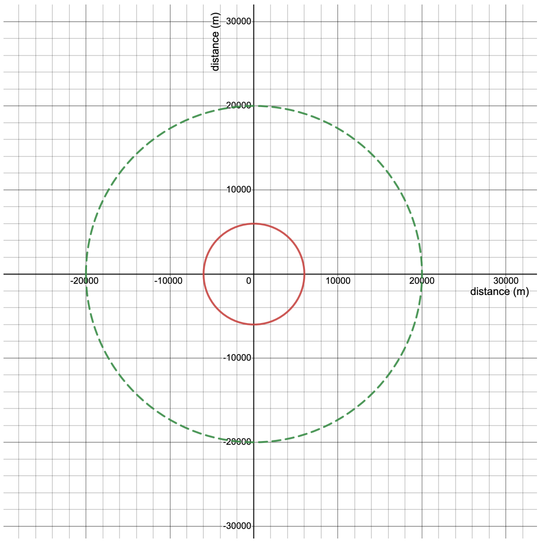


Figure 11: Radii of forager bee traveling distance from the hive.

4.4 Fitting Beehives into 20 Acres

To see the number of beehives that can fit into the plot of land, making sure that all of the land is covered, beehives were modeled onto a 2D plane. This model can be seen in Figure 12. Only one beehive is necessary to make sure all flowers on the plot of land can be reached by the bees since the circle is larger than the plot area.

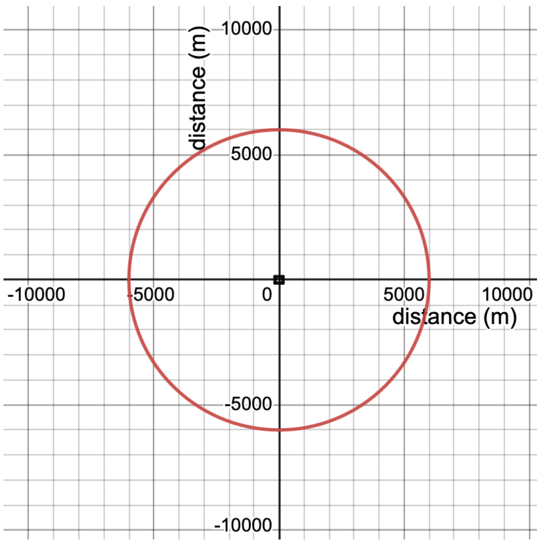


Figure 12: Plot area within the beehive region.

4.5 Flower Pollination

4.5.1 Overview

To ensure that one beehive contains enough bees to pollinate all of the flowers on the plot, we need to determine how many flowers that the bees flying within 6,000 meters of the hive can pollinate, compared to the actual number of flowers in the plot.

Furthermore, the plot of land is located within the 6,000 m beehive region, so we calculated how many flowers that the bees flying within 6,000 m are pollinating. Additionally, we assumed that every flower must be pollinated at least one bee a day.

4.5.2 Flowers Pollinated by Bees Flying Within 6,000 m of Hive

Given that a hive has 40,000 bees, we can use our hierarchy tree (Figure 1) to find the total number of forager bees, or bees who pollinate flowers: 11,760. Furthermore, according to Figure 10, 90% of foragers fly within 6,000 m of the hive. Applying this to the previous result, there are 10,584 forager bees flying within 6,000 meters of the beehive.

Next, we calculated the number of flowers that can be pollinated by 10,584 bees. Since each bee visits 2,000 flowers a day, we found that the total number of flowers the bees are visiting is 2,168,000 flowers.

4.5.3 Total Flowers in Field

To determine the total number of flowers on the plot of land, we assumed (a) there exists 10,000 plants in each acre, and (b) each plant contains 3 flowers to be pollinated. This indicates that there are 30,000 flowers per acre and 600,000 flowers on the 20-acre piece of land.

4.5.4 Comparison

There are enough bees from one hive to pollinate all the flowers in the field. Since there are 600,000 flowers in the area but enough bees to pollinate 2,168,000 flowers. Thus, only **one beehive** is necessary to pollinate a 20-acre piece of land. One beehive would ensure that all flowers can be reached and pollinated.

4.6 Generalization

As an extension to our initial solution, we wanted to analyze the optimal arrangement of hives in larger areas. We formed a plot with the same length-to-width ratio as the 20-acre plot, but with a larger area. The model would need to optimize the placement of the beehives within the plot of land to make sure that there are enough bees to pollinate all flowers and that there are no gaps between the beehive regions.

This generalization takes an approach where beehives are placed in a layout to eliminate all gaps where bees are unable to pollinate flowers. When placing the beehives on the plot, it must overlap with another beehive so all regions are reachable.

First, we placed two beehives adjacent to each other on the sides of the plot. This maximizes the amount of space covered by each beehive radius that lays within the plot. The placement of these two beehives can be seen below in Figure 13.

Unfortunately, there still exists small gaps at the top and the bottom of the plot -that are not reached by the bees. To fill these gaps, two more beehives were added to the top and the bottom of the plot of land (Figure 14).

Overall, **four beehives** are necessary in order to fill this larger plot of land. With four beehives, all flowers in the plot of land will be reached and pollinated since there is overlap. This generalizes to even larger areas of land where this arrangement can be extrapolated based on the total area.

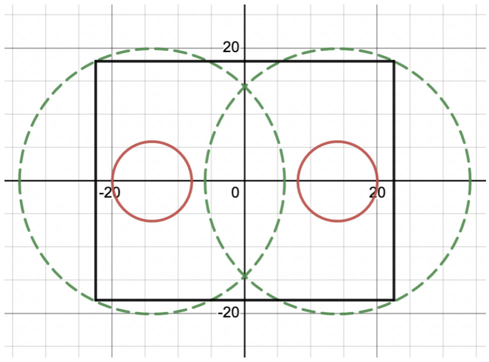


Figure 13: Amount of land covered by two beehives.

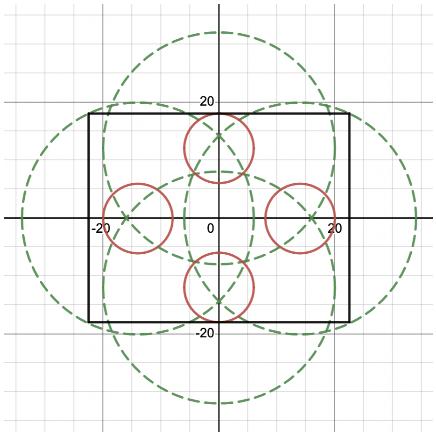


Figure 14: Amount of beehives needed to cover all the land.

5 Conclusion

5.1 Advantages of Cyclic Population Model

This cyclic model determines that the population of bees will vary between approximately 30,000 and 70,000, and this periodic nature will repeat every year. The model at hand presents many innovative and optimal rates used to find the rate of population growth, thus the population size at any given time. The most significant achievements of this population model are listed as follows.

- This model incorporates the carrying capacity as a part of the birth rate for bees to prevent overpopulation.
- This model takes into account, seasonal changes using damping trigonometric functions to model different rates in the winter and the summer.
- The model adjusts the death rates based on the current population.
- This model details the distinct differences between foraging bees and non-foraging bees and presents unique death rates to both.

5.2 Limitations of Cyclic Population Model

Although the designed cyclic model presents many unique advantages such as its periodic nature and accurate rate-based modeling, there are several limitations that influence the potential accuracy of this model in real-world analysis. These limitations are listed as follows.

- This model does not account for catastrophic events, habitat destruction, or significant environmental changes.
- This model does not consider activity, protein abundance, and pollen consumption of the bees and its influence on population size.
- The predator sub-model did not consider various different predators and their individual impacts on the bee population.
- The model did not consider different species of bees other than honey bees and their impact on the environment and their population.

5.3 Advantages of the (N)estimation Model

Our model determines that this 20-acre area will require one beehive to ensure pollination of the entire field. The designed model was generalized to larger plots of land. Furthermore, our model covers unique cases where the bee hives may fail to cover certain areas of land, and describes optimal solutions for doing so.

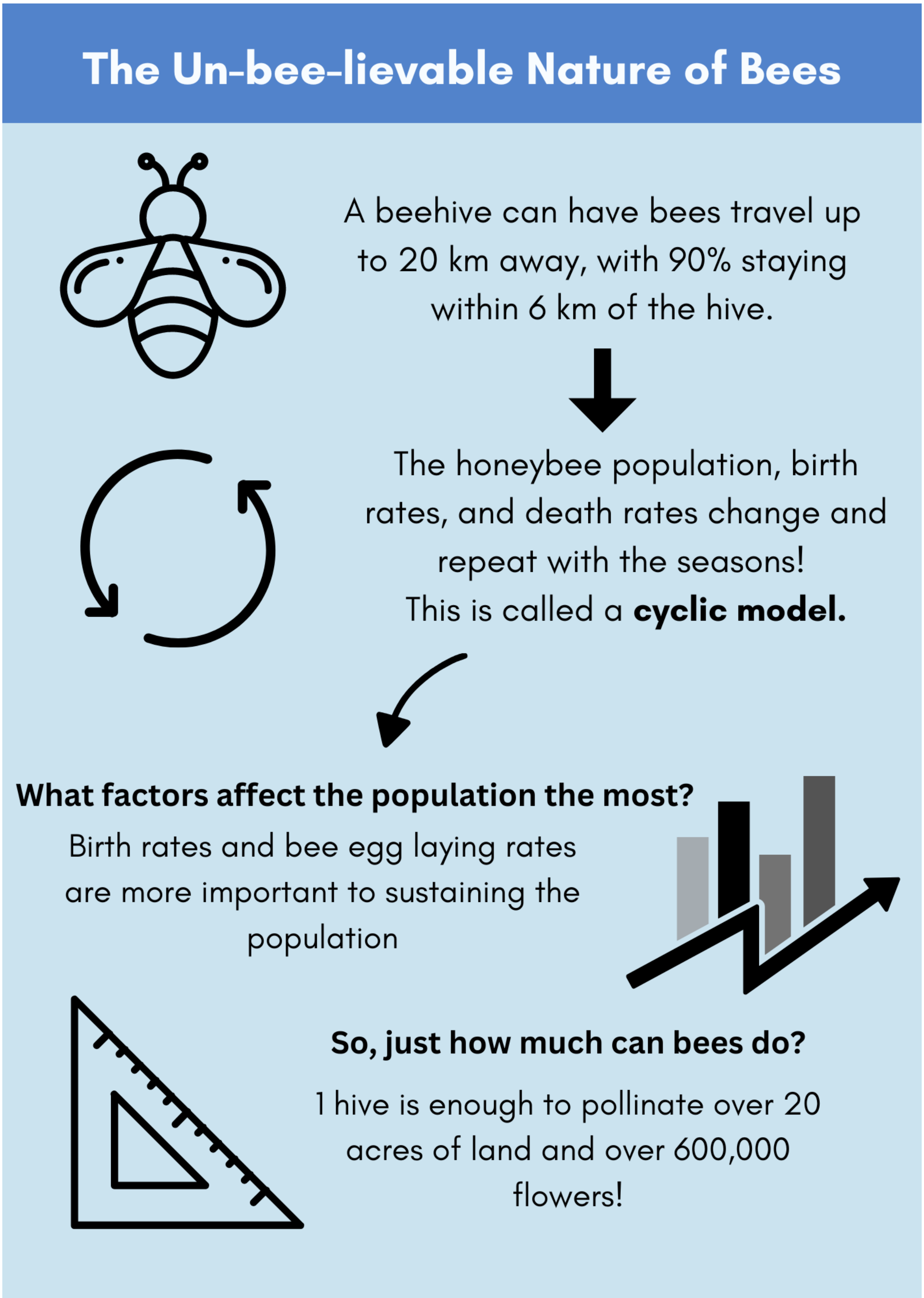
5.4 Limitations of the (N)estimation Model

Although the designed model provides accurate area-based judgment in determining the number of hives to cover a specific area, it has a few limitations that may make it difficult to generalize to different scenarios.

- The model does not account for different plants in the field which may have altered the number of flowers in each region.
- This model does not account for changing bee populations or hives with different populations.
- This model does not account for areas with an extremely high floral count.
- This model does not represent competition within the hive. Forager bees from the same colony may need to compete with each other if there is insufficient flora in the area covered by their hive.

6 Appendices

6.1 Appendix 1. Infographic



6.2 Appendix 2. Euler's Method Python Script

./himcmeulersimulate.py

```
import math
import matplotlib.pyplot as plt
import numpy as np

L = 3000 #max eggs laid per day
M = 0.98 #percentage of eggs fertlized

def dp(t, P): #t is time in years, P is current population
    birth_rate = L*M*365/2 * (80000-P) / 80000 * np.sin(2 * math.pi * t) +
        L*M*365/2
    death_rate_forager = 0.3 * M * P * (1-pow(0.7, 365*(t+1)))*((np.sin(2 *
         math.pi * t) + 1) / 2)
    death_rate_non_forager = 0.7 * M * P * (1 - pow(0.6, 365*(t+1))) * ((np
        .cos(2 * math.pi * t)+2)/2)
    predator_prey_rate = 14.7 * P + 1.764 * P * (8.3 * np.log10(0.17 * pow(
        np.exp(0.0735*P/100000) / math.sqrt(0.3 * M * P), 0.04)))
    disease_rate_forager = 0.009 * M * P * (np.cos(2 * np.pi * t) + 2)
    disease_rate_non_forager = 0.07 * M * P * (np.cos(2 * np.pi * t) + 2)
    # print(birth_rate, death_rate_forager, death_rate_non_forager,
        predator_prey_rate, disease_rate_non_forager, disease_rate_forager)
    return (birth_rate - death_rate_forager - death_rate_non_forager +
        predator_prey_rate - disease_rate_non_forager - disease_rate_forager
        )/10


def find_pop(time):
    times = []
    populations = []
    t, P = 0, 20000
    step_size = 0.1
    steps = (time-t) * int(1 / step_size)
    for i in range(steps):
        t, P = t + step_size, dp(t, P)
        times.append(t)
        populations.append(P)
    return times, populations

times, populations = find_pop(10)
print(len(populations))
print(populations[-1])
plt.plot(times, populations)
plt.xlabel("Years after 2007")
plt.ylabel("Population of Bee Colony")
plt.show()
```

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