

## Problem Choice

**E**2025  
MCM  
Summary Sheet

Team Number

**2525325**

WE ARE HERE TO MODEL the agri-"culture" and sustainability of farming practices on ecological systems. Problem E consisted of two subtasks: (1) model a food web in a farm ecosystem that replaced a previous forested area and (2) measure the impact of new species and organic farming practices on this ecological environment.

To model the ecosystem food web, we developed a network graph that shows the energy flow between predators and prey in the various trophic levels. We used *Linear Programming* and formulated an optimization problem to determine how energy is transferred between species while considering factors like how efficiently energy is passed up the food chain, the laws of energy conservation, and the energy each species uses through metabolism. To model a changing population over time, we then employed a modified *Lotka-Volterra* system of differential equations for the various predator-prey relationships, which was calibrated on the energy transfer parameters from the previous model and ecosystem characteristics. We also considered the regrowth of edge habitats using a forestation model, inspired by population growth and heat-dispersion models, and how it impacts producer populations over time. We combined these models into a single, iterative framework to predict future species populations.

After developing our model, we ran simulations on a variety of scenarios, including: (1) an undisturbed agricultural ecosystem, (2) an introduction of 2 new species over an interval following edge habitat reforestation, and (3) an adoption of organic farming practices (mainly no pesticides/herbicides). In each case, we assessed ecosystem stability, tracked changes in species populations, and analyzed how energy flowed through the food web.

From these simulations, our results show that ecological systems approach an equilibrium solution and are generally stable under most conditions. Across all regions, the biomass of producers, herbivores, and other consumers tend to follow the trophic 10% rule of thumb, where there are about 10 times more producers than herbivores, herbivores than secondary consumers, and so on. Furthermore, we found that the presence of pesticides and herbicides help regulate the seasonal cycles of producers and herbivores. This means that with pesticides and herbicides, there is less variation in herbivore and producer populations over time.

Lastly, to test the validity of our model, we employed a sensitivity analysis including a Monte Carlo sampling simulation varying initial populations by up to  $\pm 30\%$ , which revealed over the course of 100+ simulations that tertiary populations vary the most and producer populations have the most effect on other species' populations (chain effect). However, these populations had relatively minimal changes, demonstrating our model's robustness and accuracy.

Overall, our modeling approach is adaptable, allowing us to easily add new species, incorporate different disturbances, or evaluate the effects of policy changes. Looking ahead, we plan to enhance the model by including random events to simulate natural variability, using more complex equations to describe species interactions, and factoring in broader environmental influences like climate change.

**Keywords:** Network graph, Food web model, Linear programming, Lotka-Volterra System, Forestation model, Partial differential equation, Monte Carlo simulation

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# 1 INTRODUCTION

## 1.1 Background

In recent decades, the global agricultural sector has significantly transformed its production, productivity, and utilization of resources. From 1961 to 2020, the global population grew 2.6 times, and to accommodate the rising quality of life and populace, agricultural output quadrupled [12]. Accordingly, the world production of agricultural commodities has grown from \$1.1 trillion to \$4.3 trillion [12]. The increase in agricultural yields is the result of improved irrigation systems, genetic engineering of crops, and new technologies, driving down prices and making food and material widely accessible.

However, this expansion in agriculture comes at a cost: roughly 38% of the Earth's land is used for agriculture [9]. Farming relies on ample land and resources, which has led to unsustainable levels of deforestation. Each year, agribusiness, or the clearing of forests to make space for crops and livestock, leads to the loss of around 4 million hectares of forest [15]. Agriculture is the leading cause of deforestation and habitat destruction, leaving indelible marks on the surrounding ecosystem and food webs [1].

In an established ecosystem, food webs and the flow of energy between trophic levels dictate the carrying capacity and changing populations of each species. Producers are most abundant due to their ability to self-supply energy and transfer energy up the food web to consumers, which in turn supply energy to consumers in higher trophic levels. However, because biomass decreases with each trophic level, the populations of higher levels are strictly limited by the biomass of lower levels [3, 11]. As such, reduced forested habitats and the use of chemicals, such as herbicides and pesticides, from agriculture significantly change the carrying capacities of species near farmlands, threatening the balance of the original food web.

However, an increasing amount of farmland is being used for organic agriculture, with the United States reaching 4.89 million organic-certified acres in 2021 [10]. Similarly, an increasing number of farmers are shifting away from chemicals in their production. These practices reduce agriculture's negative impacts on biodiversity and wildlife habitat quality [5, 13]. Understanding the impact of agriculture on surrounding ecosystems and food webs is crucial to model its effects on the environment and the result of more sustainable agriculture.

Therefore, this paper seeks to develop a mathematical model that quantifies the impacts of agricultural expansion and practices on surrounding ecosystems and their food webs. By taking into account a wide range of factors like changes in land use, chemical inputs from fertilizers and pesticides, and shifts toward organic farming, the model will simulate how these variables influence species populations and energy flow between trophic levels. The goal is to understand the extent to which sustainable agricultural practices can mitigate negative environmental effects and to provide a predictive tool for policymakers and farmers aiming to balance agricultural productivity with ecological conservation.

To illustrate our understanding in this field, we developed the following mind map for our model (Figure 1).

## 1.2 Problem Restatement

The problem presented has the following requirements:

1. Build a basic food web model for an agricultural ecosystem that has replaced a forested region. Include producers and consumers and model how the agriculture cycle and seasons change the system over time. Consider how herbicides and pesticides affect plant health, insect populations, bat and bird populations, and ecosystem

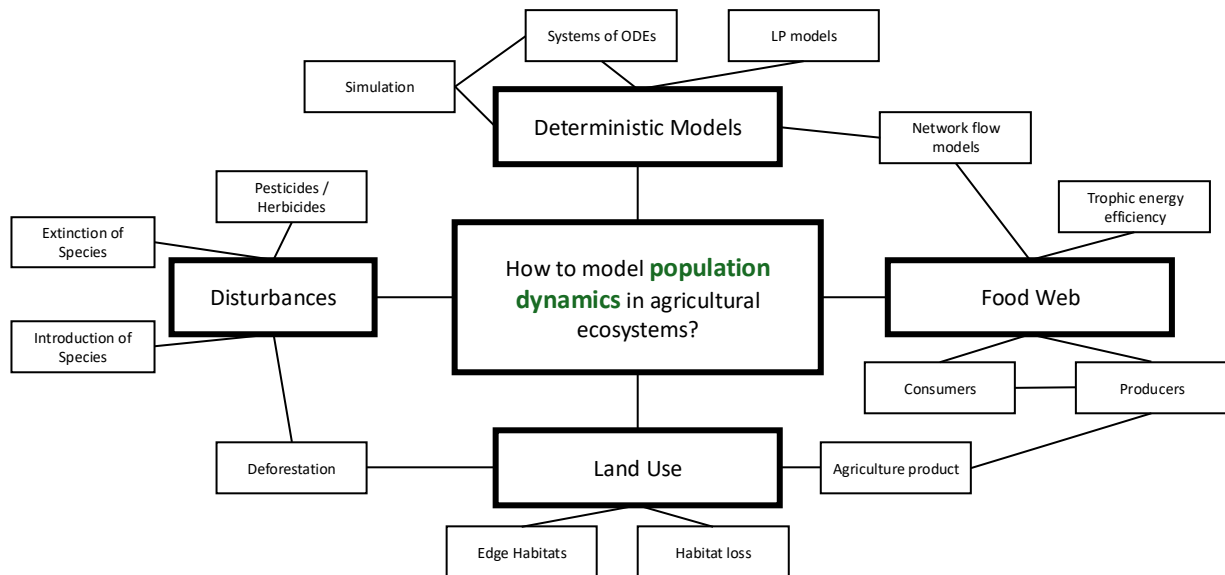


Figure 1: Mind map and brainstorm for model.

stability.

2. As edge habitats re-mature and attract native species back to the area, model how the agricultural ecosystem changes from the interactions of these species with the changed environment. Incorporate two species into the ecosystem model to determine the impacts.
3. As the ecosystem matures, report on its stability in terms of the producers and consumers if farmers removed herbicide use.
4. Analyze the effects of a farmer, who is considering organic farming methods, in different scenarios with varying components of organic farming. Demonstrate the impact on the ecosystem as a whole and its components, factoring in aspects like pest control, crop health, plant reproduction, biodiversity, long-term sustainability, and cost-effectiveness.

After developing and testing the model, we are tasked with presenting our findings to a farmer:

1. Write a one-page letter to a farmer who is exploring organic farming practices. Advise the farmer on what methods to employ, including discussions on sustainability and the economic trade-offs. Help the farmer determine strategies to balance costs and sustainability as well as advocate for certain policies which could incentivize conservation in agriculture.

While it's not explicitly stated in the problem, one must also remember that the farmer is likely not familiar with the specifics of math modeling. Thus, in writing the letter, a secondary objective is that we must also make our techniques accessible to a layman, taking care not to adopt a condescending tone.

## 2 MODELING A FOOD WEB

### 2.1 Problem Analysis

Part 1 of the problem tasks us with building a food web model for a new agricultural ecosystem recently converted from a heavily forested area. Additionally, the model should include not only producers and consumers but also the effects of the agricultural cycle and herbicide and pesticide use on plant health, insect populations, bat and bird populations, and overall ecosystem stability.

Since agriculture frequently introduces changes to the ecosystem, we use an iterative dynamic model combining an energy-transfer food web and a modified Lotka-Volterra population system to model changes in species populations and carrying capacities based on previous states.

### 2.2 Assumptions

#	Assumption	Justification
1	No major disturbances will occur.	It is impossible to account for unseen disasters. This assumption simplifies the situation so that the typical effects of agriculture can be modeled.
2	Predator-prey relationships do not change over time.	In order to model food web interactions, predators depend on their prey to survive, so this should not change during the modeling timeframe.
3	Newly deforested land and edge habitats have no forestation ( $F_{xy}$ ).	Crop agriculture requires removing existing vegetation and typically requires tilling. Therefore, no forestation should exist immediately following deforestation.
4	Agricultural land is a group of connected square plots.	This allows for easier simulation and modeling [14].
5	Organic farming causes plants to be part of the ecosystem.	We simulate everything twice to visualize the impacts of organic farming: once with crops as a part and once separate from the ecosystem.

### 2.3 Brief Overview

Below is a table of variables used throughout our model.

### 2.4 Food Web

We wanted to first mathematically define the food web as graph  $G = (S, E)$  as a set of species  $S$  and edges  $E$ . There exists an edge  $e_{ij} \in E$  if species  $i$  has some predatory ecological relationship with species  $j$ . We want to construct a food web to model the transfer of biomass and energy between species in this ecosystem.

Given initial conditions like species population ( $x_i$ ) and base energy production per each unit of species ( $E_i$ ), a food web model would be able to obtain the energy transfer from species to species which optimizes ecological stability and satisfies nature's constraints. First, we identified the constraints to the ecosystem such as energy balance and trophic flow.

#### 1. Energy balance constraint

Variable	Symbol	Description
Species	$i$	A species such as bats or crops
Population	$x_i$	Population of species $i$
Energy production	$E_i$	Energy production per unit of species $i$
Energy transfer	$C_{ij}$	Energy transfer from species $j$ to species $i$ (i.e., how much species $j$ contributes to species $i$ energy intake)
Energy loss	$L_i$	Energy loss due to metabolism and inefficiencies for species $i$
Trophic transfer efficiency	$\eta$	Trophic transfer efficiency (typically 10% in ecosystems)

For each species  $i$ , the energy input must balance the energy output (Law of conservation of energy) [7]. That is, in mathematical terms, for all species  $i$ ,

$$\sum_j C_{ij} + x_i E_i = L_i + \sum_k C_{ki} \quad (1)$$

where  $\sum_j C_{ij}$  is the energy obtained from consuming other species if edges  $e_{ik}$  exists ( $k$  consumes  $i$ ) and  $x_i E_i$  is the primary energy production. Furthermore, recall that  $L_i$  accounts for energy loss and  $\sum_k C_{ki}$  is the energy transferred to higher trophic levels only if edges  $e_{ki}$  exists ( $k$  consumes  $i$ ).

## 2. Trophic energy transfer

Using trophic transfer efficiency  $\eta$ , we want to ensure that the energy is moving up the food chain within the food web. So, for all species  $i, k$ ,

$$C_{ki} = \eta \sum_j C_{ij} \quad (2)$$

This ensures only a limited portion of consumed energy is transferred upward. Usually,  $\eta \approx 10\%$  for most ecological systems [2].

## 3. Non-negativity constraints

We want population and energy flow to be positive such that  $x_i > 0$  and  $C_{ij} \geq 0$  if  $e_{ji}$  exists ( $j$  consumes  $i$ ).

## 4. Maximum energy capacity constraint

Each species has to limit how much energy it can process, based on its population and energy needs. Therefore, for all species  $i$ ,

$$\sum_j C_{ij} \leq x_i E_i + U_i \quad (3)$$

where  $U_i$  is the upper bound on amount of energy a species can consume, which is derived by the carrying capacity of each species divided by the energy production of each unit of that species  $i$ .

Furthermore, we assumed that an ecological system is productive and is in optimal health given its situation, so we determined our objective function as to maximize the total energy transfer within the system, thereby constructing a max-flow problem in this food web [6]. Our objective function is defined by

$$\textbf{Objective: } \max \left( \sum_i \sum_j C_{ij} \right) \quad (4)$$

which encourages maximal energy circulation and robust energy distribution. However, measuring energy in terms of pure energy (Joules), is difficult since we may not know the exact amount of energy a species can produce. However, it is much simpler to measure energy in terms of biomass, which we can easily find production and consumption rates from historical data. Therefore, we can solve this maximization problem using scientific computing linear programming (SciPy) methods.

To summarize, we formulated a linear programming optimization model with inputs **initial population**, **energy production**, and **food web digraph**, which optimizes and outputs the **flow of energy** between species. These flow of energy outputs are then used in a Lotka-Volterra system to model the change in species populations after some specified time. Our model can be written as

$$\begin{aligned} & \text{maximize } \sum_i \sum_j C_{ij} \\ & \text{subject to } \sum_j C_{ij} + x_i E_i = L_i + \sum_k C_{ki} \\ & \quad C_{ki} = \eta \sum_j C_{ij} \\ & \quad \sum_j C_{ij} \leq x_i E_i + U_i \\ & \quad C_{ij} \geq 0 \end{aligned}$$

## 2.5 Food Web Perturbations

The optimization model above identifies the flow of energy and population interactions (predation rates) between species. We constructed this model with the aim of easily modeling environmental perturbations. Such perturbations may include: human farming, harvestation, habitat loss, introduction/loss of species, and large changes in native species populations. We describe the process of modeling these perturbations in our simulations below.

- Human agriculture / harvesting season

The food web model can be used to model the impacts of human agriculture / harvesting the land by altering energy production rates ( $E_i$ ) and adjusting the carrying capacity constraints ( $U_i$ ) to reflect habitat loss or resource depletion. This helps our model simulate how agricultural expansion, overharvesting or deforestation impact energy transfer and the health of an ecosystem.

- Introduction and extinction of species

If a new population is introduced to the environment, we can rebalance the ecosystem energy transfer by including a new species with its initial

population, energy production, and relationship to all other species. If a species goes extinct, we can update the food web accordingly and re-optimize the energy transfer to recalibrate our system.

- Changes in species populations

In the next section, we further discuss how we use a Lotka-Volterra system of differential equations to model changes in population. We improve this system by iteratively balancing the ecosystem with our LP model, then simulating Lotka-Volterra for a short amount of time. In this way, we can better be able to account for long-term changes to the ecosystem.

## 2.6 Lotka-Volterra Population Model

To model the population of different species and trophic levels over time, we use the Lotka-Volterra system. The Lotka-Volterra system is a set of nonlinear differential equations modeling the dynamics of interacting populations, most notably in predator-prey relationships, which fits perfectly in our scope of modeling an ecological food web. The system consists of two coupled equations: one governing the growth of the prey population, which experiences exponential growth in the absence of predation, and another governing the predator population, which depends on the availability of prey for sustenance. These equations capture oscillatory behaviors characteristic of ecological systems, where predator populations lag behind prey populations in cyclic fluctuations.

In our use case, we use a generalized form of Lotka-Volterra for  $n$  species, as new species may be added or removed based on an introduction or extinction event. Mathematically, we let there be  $n$  native species, where the population of a native species  $i$  (under the conditions  $1 \leq i \leq n$ ) is denoted by  $x_i$ . Therefore, the rate of change of a population *in relation to* other species can be represented as

$$\frac{dx_i}{dt} = r_i x_i \left( 1 + \frac{\sum_{j=1}^n \alpha_{ij} x_j}{K_i} - d_i \right) \quad (5)$$

where  $r_i$  is the uninterrupted growth rate of species  $i$ ,  $\alpha_{ij}$  is the consumption rate of species  $j$  on species  $i$ ,  $K_i$  is the carrying capacity of species  $i$ , and  $d_i$  the mortality rate of each species. The parameters  $\alpha_{ij}$  can be combined into an  $n$  by  $n$  matrix of "interaction" parameters. For example, if  $\alpha_{ij} < 0$ , then it means that species  $j$  harms species  $i$ , and vice versa for  $\alpha_{ij} > 0$ . Therefore, we denote this matrix of parameters the *interaction matrix*. For these  $n$  species, the Lotka-Volterra equations will yield a system of  $n$  differential equations with an  $n$  by  $n$  matrix  $\alpha$ .

However, death rate ( $d_i$ ) is more complicated than just natural mortality rate. Factors such as herbicide and pesticide use, GMO techniques, and synthetic fertilizer use (inorganic farming), all contribute greatly to the death rates of producers [8, 13]. To distinguish organic farming from non-organic farming, we assume that organic farming means crops become a part of the ecosystem since there are no pesticides/herbicides used to keep consumers away. This would therefore increase  $d_i$  such that it incorporates the consumption of crops by consumers.

Note that since we balanced the ecological food web in the previous section, we can use those energy transfers to derive the interaction matrix parameters of the Lotka-Volterra system without the need for estimating parameters. These interaction parameters can also



be viewed as predation / consumption rates in context. In practice, we estimate  $\alpha_{ij}$  as the following ratio of energy flow to total energy produced.

$$\alpha_{ij} = \frac{C_{ij}}{\sum_k C_{kj}} \quad (6)$$

See the following example interaction matrix for four species in an ecological system.

$$\alpha = \underbrace{\begin{array}{l} \text{Species 1} \\ \text{Species 2} \\ \text{Species 3} \\ \text{Species 4} \end{array} \left[ \begin{array}{cccc} 1 & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,4} \\ \alpha_{2,1} & 1 & \cdots & \cdots \\ \alpha_{3,1} & \cdots & 1 & \cdots \\ \alpha_{4,1} & \cdots & \cdots & 1 \end{array} \right]}_{4 \text{ species}} \quad (7)$$

When we introduce new species, the Lotka-Volterra model will yield a system of  $n + 1$  differential equations with an interaction matrix  $\alpha$  of dimensions  $n + 1$  by  $n + 1$  where all values  $\alpha_{ii} = 1$  for self-interaction of species.

Similarly, the balance of energies to reach equilibrium in the food web network model provides the carrying capacity  $K_i$  for each species  $i$  in the ecosystem based on the total biomass, in kilograms, that can be transferred from all of its prey. So, to convert the maximum possible biomass supported to a population, for each species  $i$ ,

$$K_i = \frac{\sum_j C_{ij}}{M_i} \quad (8)$$

where  $C_{ij}$  is the energy, or biomass, transferred from each prey  $j$  to species  $i$ , and  $M_i$  is the average mass, in kilograms, of a member of the species. It is important to note that carrying capacities apply to the consumers in the ecosystem, but as autotrophs, producers are not constrained by limits on energy. Rather, their population dictates the populations of their consumers, meaning that excessive growth or decline will be offset by opposite changes in consumer populations.

Given an initial state  $\mathbf{P}$  and an interaction matrix  $\alpha$ , we can simulate all species populations over a short-period of time. After a time-step (one-season), we can recalibrate our ecological system with the food web optimization model based on any human decisions and environmental changes.

A full visual of how the population and food web model work together to simulate the ecological system in a specified environment is shown below in Figure 2.

## 2.7 Adjusting for Edge Habitats

To account for edge habitats and how they impact the existing ecosystem, its growth must first be modeled. Here, we introduce a variable  $F \in [0, 5]$  to measure the *forestation level* of a particular area, with 0 meaning no vegetation and 5 meaning heavily forested. Each area of the agricultural ecosystem will have or support a population of producers proportional to its forestation level, meaning a level of 1 will accommodate 20% of the population supported by a level of 5.

However, for forestation level to be applied, the agricultural land and its surrounding forested area must be divided into smaller sections. Therefore, we represent the given agricultural area as a square grid  $\mathbf{A}$ , with each cell  $(a_{xy})$  represented by a forestation level in a matrix. From Assumption 4, the agricultural lands have no forestation ( $F = 0$ ). On the other hand, the remaining forested grids are heavily forested, suggesting values between 3 and 5. A sample representation of an area is shown below in Figure 2.

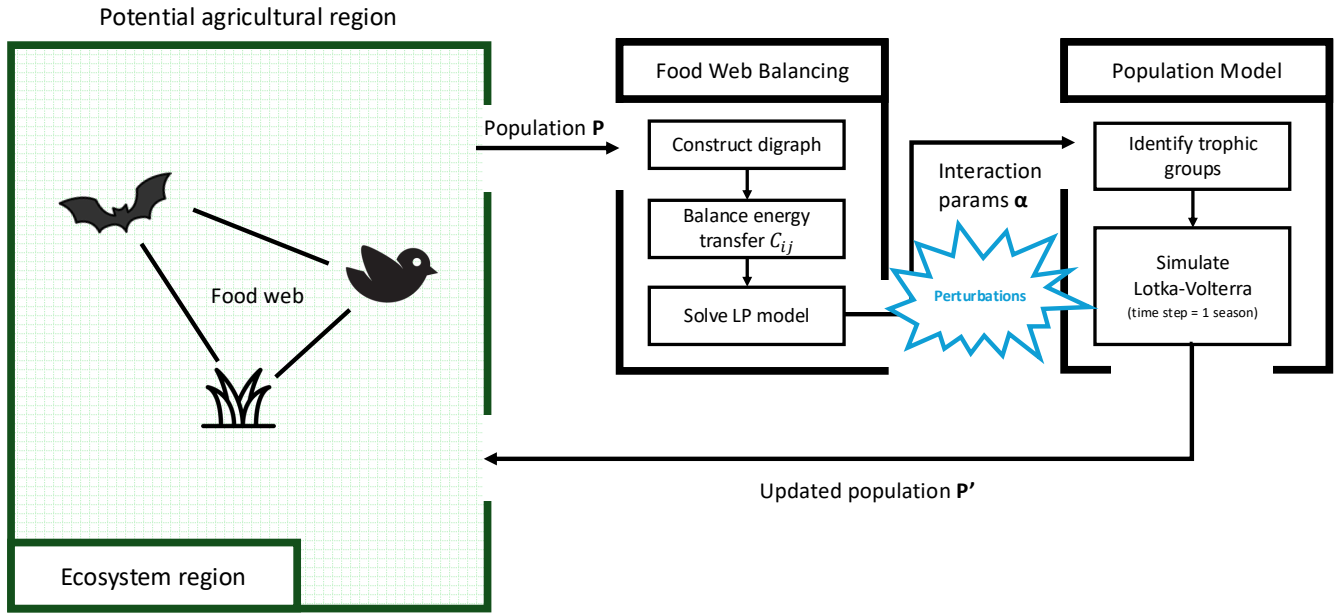


Figure 2: **Overview of ecosystem model.** Combined network analysis with Lotka-Volterra as an iterative model.

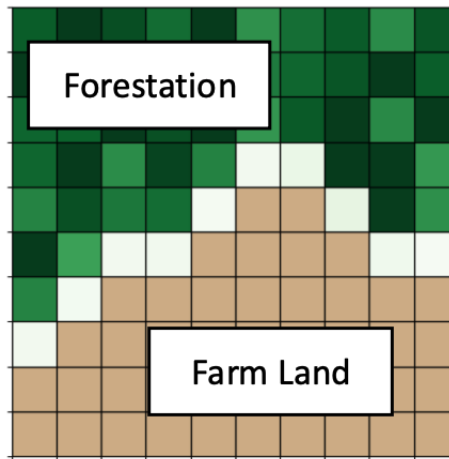


Figure 3: **Example choropleth map for area.**

Let  $F_{xy}$  be the forestation level in cell  $a_{xy}$ . In this representation, edge habitats are defined as cells that have no forestation, or  $F_{xy} = 0$ , but border a forested cell where  $F_{x'y'} > 0$ . Given this classification, the increased maturity in the edge habitat ecosystem can be modeled as the increasing forestation levels in edge habitat cells, like the heat equation PDE.

Since the edge habitat areas have been deforested, similar to the effects of a natural disaster, our model for reforestation is inspired by ecological succession and the heat equation. Therefore, an edge habitat's forestation level  $F$  changes as function of spatial diffusion of forestation and the forestation levels of its adjacent cells. First, to model growth attributed to the cell's own forestation level, we apply a partial differential equation of the form in Equation 9.

$$\frac{\partial F_{xy}}{\partial t} = \frac{r}{K} \cdot \frac{\partial^2 F_{xy}}{\partial x^2} \quad (9)$$

where  $r$  is the growth rate of the forest per season,  $F_{xy}$  is the forestation level of cell  $a_{xy}$  at time  $t$ , and  $K$  is the carrying capacity or maximum forestation level. Since the maximum possible forestation level is 5,  $K = 5$ . Furthermore, while complete forest regrowth does not fit within a feasible time-frame, faster-growing producers like grass grow significantly faster, so forestation levels will be determined principally from the grass population. While grass can reproduce every year, it requires optimal conditions, and only a small portion of buds successfully develop [4]. Thus,  $r = 0.25$  was empirically determined to best model population growth.

In addition to producer reproduction in edge habitats, population increases can be attributed to reproduction from nearby forested cells which increases plant populations in adjacent areas. The amount contributed by adjacent cells directly relates to the producer population, or the forestation level, so the increase in the edge habitat cell's forestation level can be modeled as a discrete version of the previous PDE as shown in Equation 10.

$$\frac{\partial F_{xy}}{\partial t} = \frac{r}{K} \cdot \frac{\sum_{(a,b) \in \{(x \pm 1, y \pm 1)\}} F_{ab}}{4K} \quad (10)$$

Intuitively, at time  $t = 0$ , when  $F = 0$  for an edge habitat cell, its initial reforestation will stem from adjacent cells since it has no producer population yet (an empty slate cell). A choropleth map (forestation map) of an agricultural area's edge habitat reforestation using this growth equation is shown below in Figure 4.

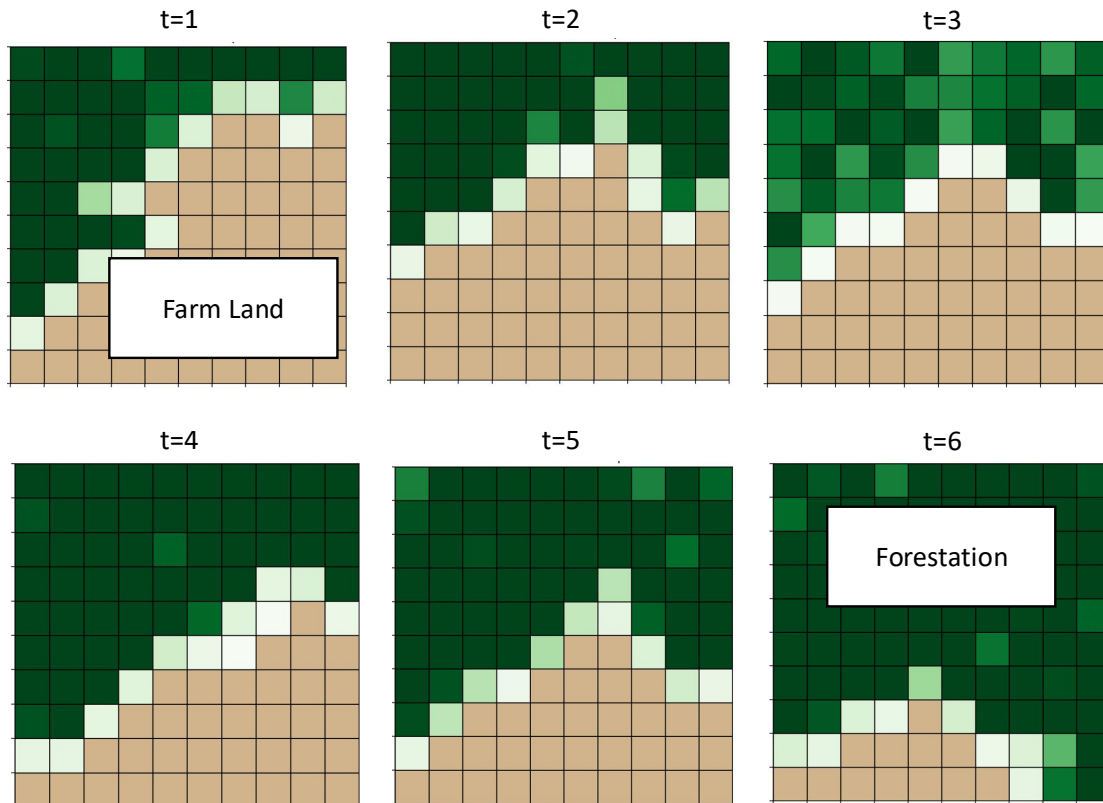


Figure 4: **Example forest growth over time.**

To incorporate this into the ecosystem model, we simulate an increase in the forestation level in each edge habitat cell, then update the producer population based on new forestation levels. To establish a conversion between forestation levels and producer biomass or population, we set a proportional relationship between the total forestation level of all

cells and the total producer population when the ecosystem is first established at time  $t = 0$ . More explicitly,

$$\sum_{a,b} F_{ab,i} = \lambda P_i \quad (11)$$

where  $F_{ab,i}$  is the initial forestation level for cell  $a_{ab}$  in the ecosystem,  $P_{\text{producer}}$  is the initial population of producers, and  $\lambda$  is the conversion factor between forestation level and population. By solving the equation, a change in forestation level can be converted into a proportional net change in the producer population. Thus, following every season, the increase in producer population in the ecosystem from edge habitats is equal to

$$\frac{\partial P_{\text{producer}}}{\partial t} = \frac{1}{\lambda} \sum_{a,b} \Delta F_{ab} \quad (12)$$

Then, we continue to the next time step with the initial producer population changing, updating the energy flow within the entire ecosystem.

## 2.8 Modeling Returning Species

We also want to model the impact of reintroducing two native species after edge habitats mature as part of the problem statement. To make sure these edge habitats mature, only habitats with high forestation levels ( $F_{xy} > 3$ ) will be livable for these native species.

Following the native species' reintroduction, the food web network model will be adjusted to maximize and optimize energy transfer. The model thus provides new carrying capacities and species interaction coefficients, which will then be used in the Lotka-Volterra population model to determine the effects on the ecosystem. An example of native species reintroduction will be shown in the next section.

## 3 APPLICATION OF FOOD WEB MODEL

To apply our food web model to a sample ecological environment impacted by human agricultural practices, several scenarios must be considered in order to analyze the impacts of natural processes and human decisions in ecosystem populations. Here, we present three main scenarios:

### 1. No major disturbances occur while the agricultural ecosystem matures.

This will provide a baseline example of expected species interactions, given the seasonal agricultural cycle and use of herbicides and pesticides in agriculture. We will analyze the equilibrium and stability of the population system and show the behavior of species populations over time.

### 2. 2 new native species are introduced in the process of maturation of the agricultural ecosystem.

This will allow us to more easily visualize the food web re-balancing process when 2 new species are introduced. We will again identify the equilibrium state and stability of the population system and show the behavior of species populations over time.

### 3. Farming practices become organic, preventing the use of GMOs and pesticides or herbicides during the agricultural process.

This means that crops now are not protected by herbicides, pesticides, or any other artificial mechanisms. Therefore, the crops become essentially part of the food chain. Therefore, we include crops as a part of the food web's producers and again simulate the behavior of species populations and the health of the ecosystem over time.

### 3.1 Undisturbed Ecosystem

Our initial food web is displayed below with producers and several groups of species (for simplicity in the model). We then run our LP model and the calibrated predation rates are shown below. We estimate the initial population by biomass instead of population count for simplicity, and model energy transfer as the transfer of biomass between species and trophic levels. We show these results and the model food web in Figure 5.

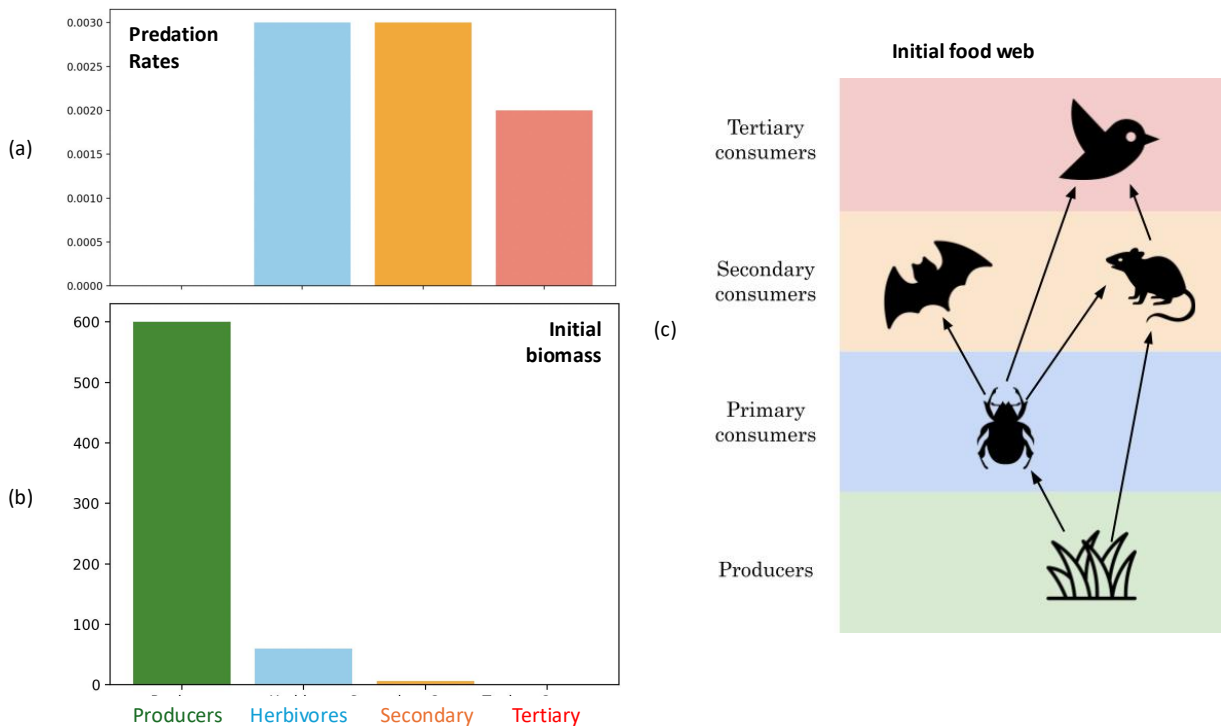


Figure 5: **Food web and initial calibrated conditions.** Note that (a) biomass is in kg and (b) predation rates are calibrated by the LP model.

As shown in the figure, the initial food web consists of grass as a producer, bugs as a primary consumer, bats and mice as secondary consumers, and birds as a tertiary or apex consumer. Additionally, the predation rates for tertiary consumers optimized from the food web network model follow expectations. Since tertiary consumers have the smallest total biomass and the least abundant food source, they should have lower predation rates.

After initially calibrating the food web energy transfer model, we then simulate the Lotka-Volterra system with the new interaction parameters  $\alpha_{ij}$ . We obtain the following population chart shown in Figure 5.

As shown in the simulation, following six seasons, the biomass of producers and consumers gradually tend toward an equilibrium where each trophic level has approximately 10% of the total biomass below it, which follows our expectations of the trophic energy transfer rule. Furthermore, the effects of one trophic level's population change on the rest of the trophic levels is intriguing. At the beginning of the first season, the producer population was excessive, making it an abundant and reliable food source for primary producers.

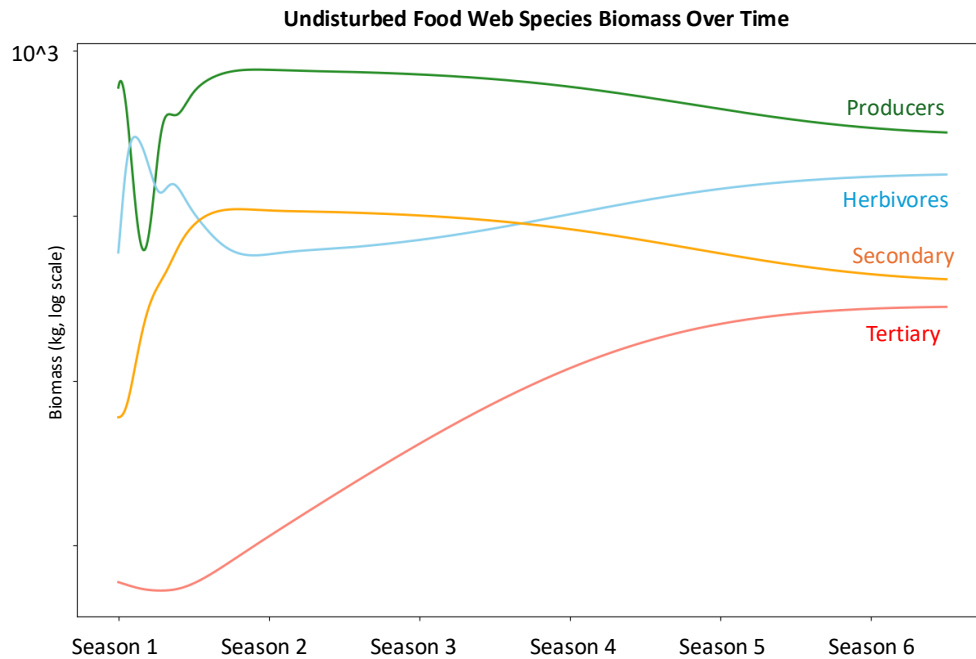


Figure 6: **Population of trophic levels (biomass) over time.**

Thus, its population declined while primary consumers' populations increased. These effects similarly affected the secondary consumer and tertiary consumer populations, which benefited from increases in prey availability. However, the increase in tertiary consumers was slowest, which is intuitive since the change in producer population would have to affect the primary and then the secondary consumers.

### 3.2 New Species Ecosystem

In this ecosystem, edge habitats gradually reach higher levels of forestation and thus increase the producer population, as outlined in previous sections. As such, after the edge habitats have reached sufficient forestation levels, two species will be gradually reintroduced over several seasons. In our test scenario, we reintroduced clovers as a producer and rabbits as a consumer that preys on grass and clovers. The effects on the species populations are shown below in Figure 7 and 8.

While the diagram reflects the updates to the food web, it is understandable that the initial biomass and predation rates are unchanged. Since the two new species are not introduced until later in the simulation, the initial ecosystem remains the same as in the undisturbed system.

As shown in Figure 8, beginning in Season 2, a population of clovers and rabbits began to be introduced into the ecosystem. This change substantially impacted the trajectories of the species populations, which is most clearly attributed to changes in primary consumers. Instead of a continued decrease in primary consumers, the gradual introduction of rabbits leveled and subsequently increased their population. This addition clearly affected the other trophic levels, which aligns with expectations: despite increases in clover populations, producer populations remained relatively constant compared to their sharp increase in the undisturbed system. Similarly, secondary consumer populations began decreasing later due to the reintroduction of primary consumers. Most notably, after six seasons the trophic level populations remain in an unbalanced, unstable state, suggesting that more time is required for the hybrid ecosystem to reach equilibrium following the reintroduction of native species.

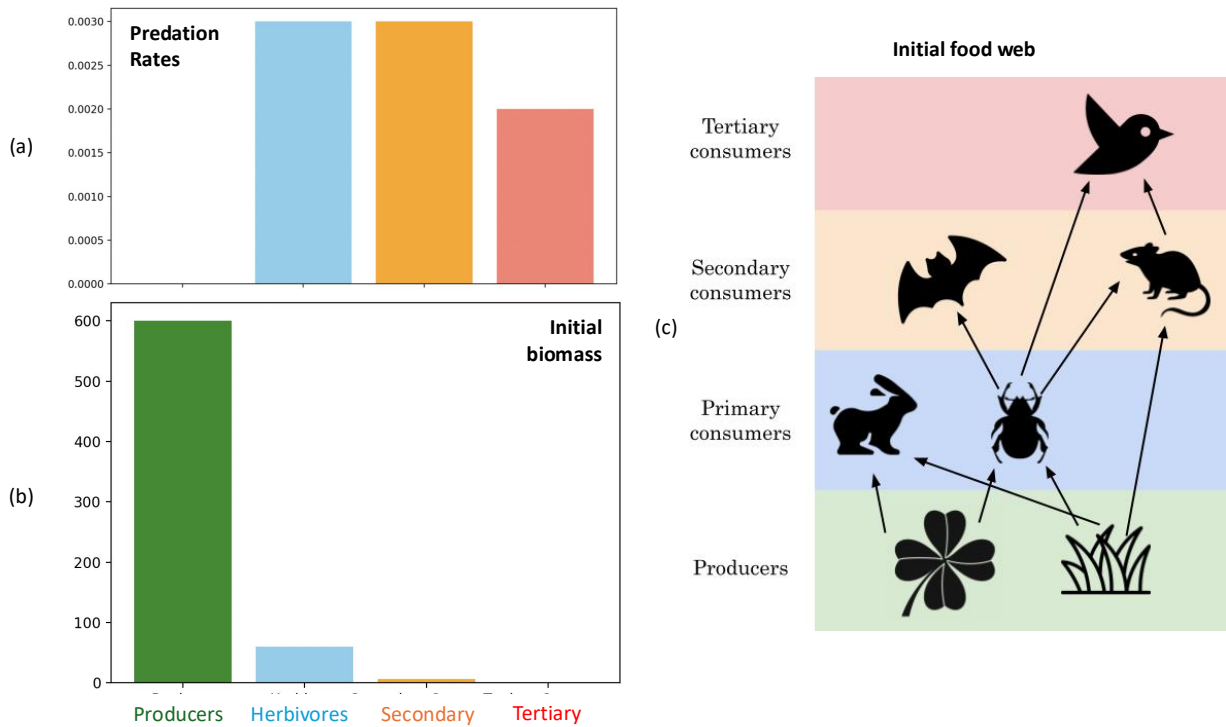


Figure 7: **Food web and initial calibrated conditions.** Note that (a) biomass is in kg and (b) predation rates are calibrated by the LP model.

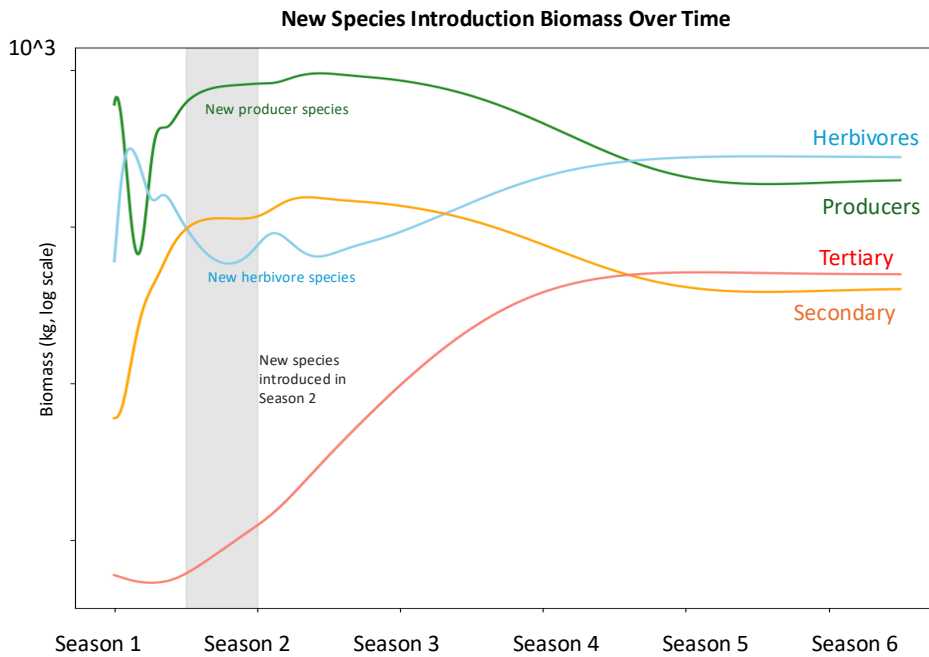


Figure 8: **Population of trophic levels (biomass) over time.**

### 3.3 Organic Farming Ecosystem

For an organic farming scenario, several changes impact the dynamic of the agricultural ecosystem. Most notably, in the absence of herbicides, pesticides, and chemicals, crops now become vulnerable to pests. Hence, they effectively become part of the ecosystem's food supply, functioning as additional producers in addition to the population of producers from previous sections. Furthermore, these crops, by definition of being "organic," cannot be

genetically modified or altered using hormones, which will reduce their yield and growth speed. Results from relevant research of all GMO crops found that genetic modification has increased crop yields by 22% [8].

The introduction of crops into the ecosystem therefore means that its growth must be modeled. While crop growth is similar to that of producers in the original ecosystem, it cannot reproduce due to being harvested every year. Rather, every 4 quarters, its population will be reset to its carrying capacity due to replanting. Crop growth can thus be represented as an increasing forestation level as the crops mature and form an agricultural forest. While the crops do not reproduce during the agricultural cycle, they gain mass, which can be represented as a logistic increase in the total biomass of the population.

At the same time, because the crops are a food source in the ecosystem, their forestation level is adversely affected by primary consumers. As such, the increase in forestation level each quarter, when not being reset, follows similarly to the Lotka-Volterra system defined in Part 2. To account for the decrease in yield, the mass of a crop plant will be decreased by approximately 18% compared to standard crop masses (equivalent to GMO crops yielding 22% more mass than non-GMO). Finally, to account for the removal of herbicides, producers no longer have an additional death rate due to agricultural chemicals.

Thus, factoring in these changes, the effect on the species in the ecosystems was simulated, as shown in Figure 10 below.

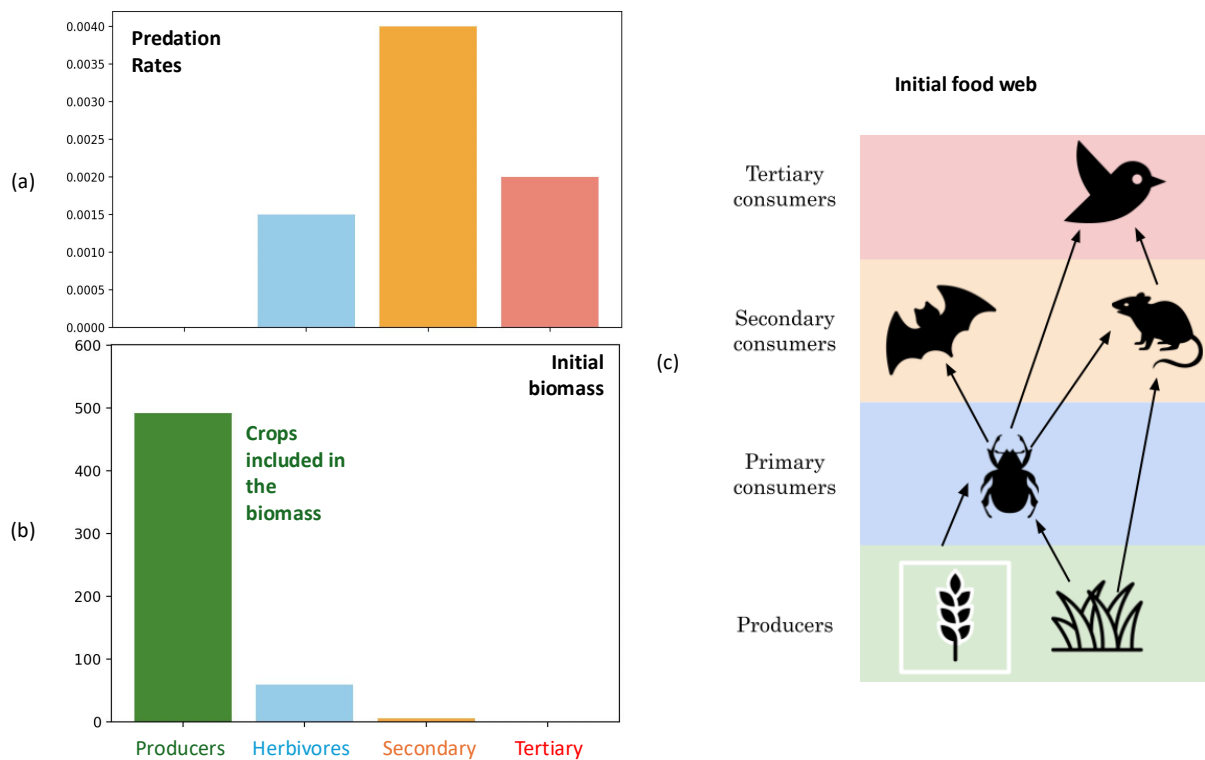


Figure 9: **Food web and initial calibrated conditions for organic web.** Note that (a) biomass is in kg and (b) predation rates are calibrated by the LP model.

As shown in Figure 9, the inclusion of organic agriculture introduced wheat as a new producer. To test the adjustment of the predation rates, the organic scenario had a smaller total of biomass, while levels of primary, secondary, and tertiary consumers remained the same as in the previous scenario. In contrast to the undisturbed scenario, the predation rate for primary consumers decreased while the predation rate for secondary



consumers significantly increased. This change can be attributed to an initial imbalance of populations since primary consumers now far exceed the approximate 10% of producer biomass equilibrium. Therefore, the excessive population of herbivores leads to more competition for producers and less competition for their predators, which explains the lower primary consumer predation rates and higher secondary consumer predation rates.

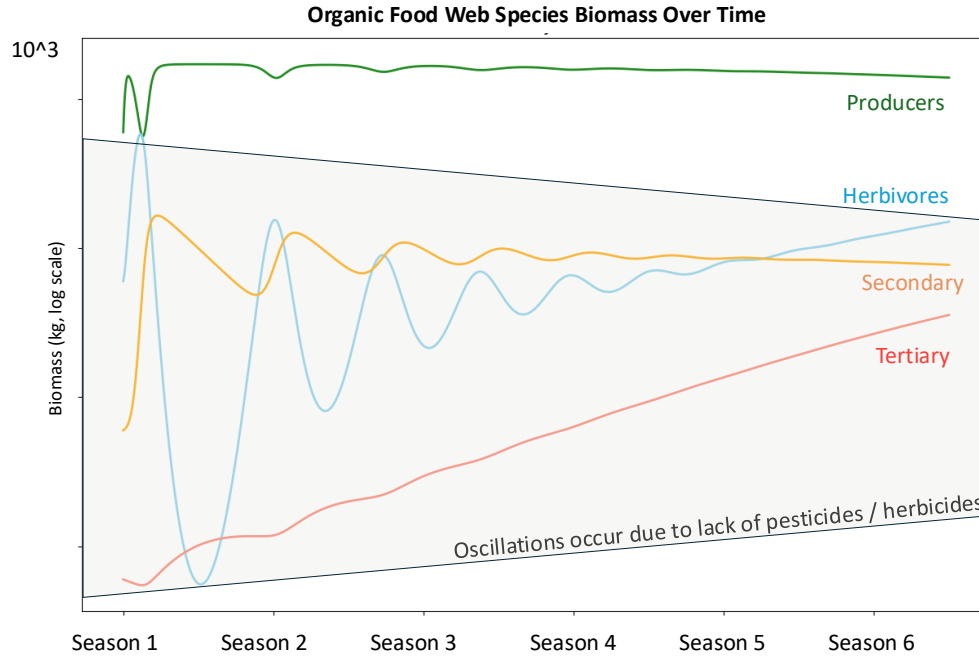


Figure 10: **Population of trophic levels (biomass) over time in organic environment.**

In contrast to the undisturbed scenario, the graph of the species populations displays a significantly higher level of oscillation, with the exception of tertiary consumers. This can be principally explained by the removal of herbicides and pesticides, which in previous scenarios dampened the level of volatility by preventing species, particularly producers and primary consumers, from significantly exceeding their carrying capacities.

Interestingly, producer populations tend to stay approximately level, which is intuitive since the growth rate of producers, including the crops, helps to offset consumption from herbivores. However, the populations eventually approximate toward equilibrium based on trophic level energy transfer and limits.

## 4 MODEL DISCUSSION

### 4.1 Stability Analysis

To identify the stability and health of this ecosystem, we employ equilibrium analysis on the Lotka-Volterra system. To identify the equilibrium solutions of a system of differential equations, we first identify equilibrium points by solving the  $n$  species system where

$$\frac{dx_i}{dt} = 0$$

Then, we must classify each of these equilibrium points based on the behavior of the system around them. Recall from differential equations analysis that the Jacobian of the system allows us to classify each equilibrium state as stable, unstable, or semi-stable.

The Jacobian can be estimated by analyzing the first-order derivatives around an equilibrium point, which are provided by the Lotka-Volterra model. The stability of a system near a stationary point can be determined by examining its eigenvalues. If all eigenvalues have negative real parts, the system is stable, meaning it will return to the stationary point after a small disturbance. Conversely, if even one eigenvalue has a positive real part, the system is unstable, and a small disturbance will cause it to move away from the stationary point.

From both a graphical (Figure 6 and 10) and Jacobian analysis of the system, the ecological system is **stable** for both organic and undisturbed scenarios and **unstable** for the new species scenario.

## 4.2 Sensitivity Analysis

Next, to examine the effects of varying input parameters on population graphs, we employ a Monte Carlo sampling simulation approach. We iteratively sample  $\pm 30\%$  of the values of the initial population setup for the undisturbed ecological scenario. We rerun the entire simulation based on these altered initial conditions. A robust model would mean that these small changes should not significantly alter the equilibrium states and overall population growth.

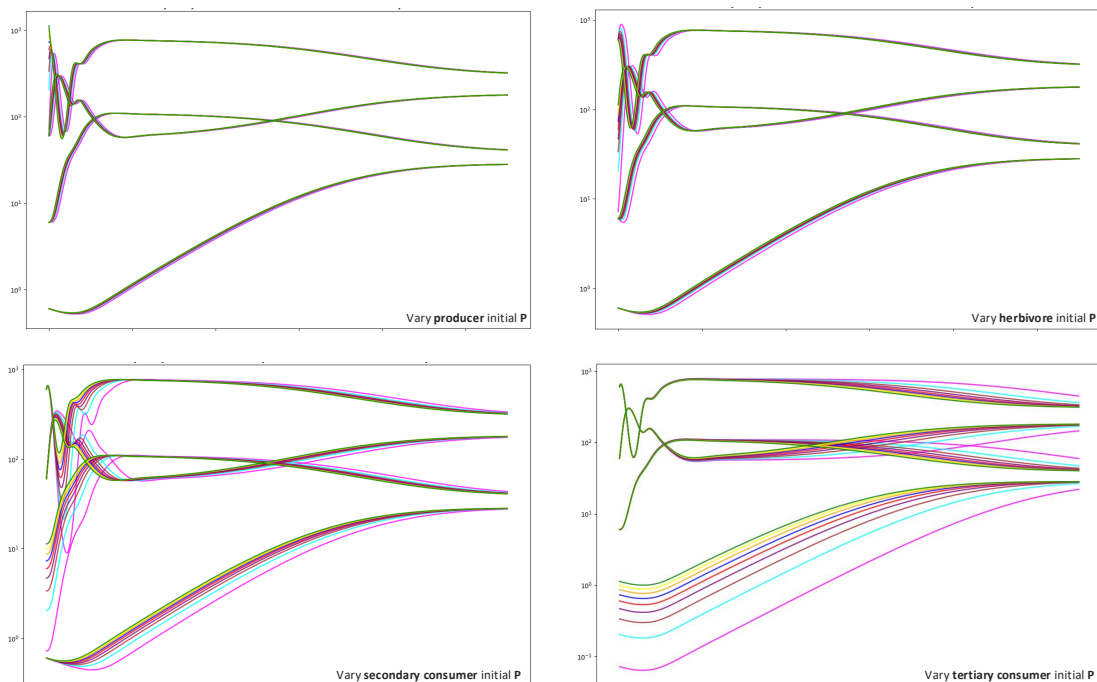


Figure 11: **Sensitivity analysis of undisturbed populations.** Varied producer, herbivore, and consumer populations separately.

The sensitivity analysis reveals several encouraging aspects of our food web model, demonstrating both mathematical stability and biologically plausible dynamics across a wide variety of parameter ranges. The model captures key ecological principles while maintaining computational tractability.

A particularly strong feature is the model's representation of producer dynamics, which shows surprising stability and resilience to initial conditions—a pattern that closely mirrors natural systems. The rapid convergence of different trajectories at the producer level, regardless of initial perturbations, suggests our implementation of density-dependent growth and herbivory effectively captures the self-regulating nature of primary producer populations.

The model demonstrates stable long-term behavior across all trophic levels, with solutions consistently converging to well-defined equilibria. This stability is particularly noteworthy given the complexity of the four-level trophic system. The intermediate responses of herbivores and secondary consumers display classic predator-prey oscillations before reaching stable states, reflecting well-documented patterns in natural systems.

At the same time, the model shows heightened sensitivity to initial tertiary consumer populations, which is above the empirically proven baseline in ecology. However, this could actually represent a feature rather than a limitation, capturing the documented cascading effects that apex predator populations can have on food web structure. However, future refinements might moderate this sensitivity to better align with observations in the life sciences.

The consistent mathematical behavior across widely varying initial conditions ( $\pm 30\%$  from normal at its peak) demonstrates the model's robust structure. Even in cases of significant perturbation, the system maintains biologically reasonable boundaries and eventually stabilizes, suggesting that our core equations effectively capture the fundamental mechanisms of trophic interactions.

### 4.3 Strengths

1. Our model's main strength is the combination of static (food web) and dynamic (Lotka-Volterra) systems.

Our model effectively combines a static food web (energy transfer) model with a dynamic population model (Lotka-Volterra), which allows for better parameter calibration and a more realistic simulation of ecosystem dynamics. The iterative coupling of these systems allows for feedback between energy flow and population changes. Furthermore, the method of deriving Lotka-Volterra interaction parameters ( $\alpha_{ij}$ ) from the energy transfer model  $C_{ij}$  avoids the subjective process of estimating these parameters independently. Similarly, using the food web model to derive carrying capacities provides a more ecologically grounded approach.

2. Our model uses biomass as an approach.

Using biomass as a proxy for energy simplifies the model and makes it more practical, as biomass data is often more readily available than precise energy measurements. Furthermore, biomass allows us to calibrate our models to virtually any ecosystem as long as estimates are provided about initial conditions.

3. Our model can easily handle disturbances such as extinctions and habitat changes.

As described in previous sections, we modeled the effect of introducing new species and updating forestation as seasons pass. Our food web LP and Lotka-Volterra models easily integrate new species, as they were designed with an arbitrary number of species in mind. Furthermore, our model can naturally represent organic farming practices since crops can be added to or removed from the food web based on the use of (in)organic farming practices.

### 4.4 Limitations

1. Our food web may not capture more nuanced relationships.

Our food web model, while robust, makes some simplifying assumptions. For example, we assume constant predator-prey relationships (Assumption 2). In reality, species can adapt their diets or hunting strategies over time. We do try to emulate this by iterating and recalibrating our food web every season. However, future work could incorporate adaptive foraging or switching behavior.

2. Our model is deterministic, and therefore lacks stochasticity.

Our model is deterministic, meaning it produces the same output given the same inputs. Real ecosystems are inherently stochastic, with random events influencing population dynamics and energy flow. Incorporating stochasticity could make the model more realistic. Furthermore, we could integrate other important environmental factors, such as climate change, pollution, and disease.

## 4.5 Future Work

To improve our model, we would explore the relationships between energy transfer and predation rates to see if nonlinear relationships exist. Furthermore, we intend to estimate the impact of parameters such as pesticide and fertilizer use with more specificity. This might include investigating soil health or toxicity and incorporating it into the ecosystem food web. The population dynamics could be also enhanced with stochasticity, the Allee effect (the theory that larger populations are stronger), and potentially agent-based modeling for species dynamics.

Additionally, the edge habitat model could be improved by using more complex diffusion, considering competition and facilitation among plants, and including the influence of consumers and predators. Lastly, incorporating external factors like climate change, pollution, and land use change would also be important for a farmer to consider.

## 5 CONCLUSION

Our paper developed a model to analyze how agricultural practices affect local ecosystems, combining food web dynamics, population modeling, and edge habitat growth. Through Linear Programming and modified Lotka-Volterra systems, we captured both energy flows and population changes across trophic levels.

Our model revealed several key insights. Ecosystems consistently reached stable equilibria across different scenarios, with biomass distributions following the expected trophic 10% rule. Pesticide use notably dampened population cycles in lower trophic levels, while organic farming showed distinct patterns in species variability. Our sensitivity analysis, varying initial populations by  $\pm 30\%$ , demonstrated the model's robustness while highlighting the particular vulnerability of tertiary consumers to system changes.

These findings have practical implications for agricultural management. The model suggests that while organic practices support biodiversity, they may require careful monitoring of population fluctuations. Edge habitat restoration proved beneficial for ecosystem stability, particularly when combined with gradual species reintroduction.

Looking ahead, this framework provides a foundation for more sophisticated modeling. Future iterations could incorporate climate variables, stochastic events, and adaptive species behavior. Most importantly, our model offers a practical tool for balancing agricultural productivity with ecosystem preservation, allowing farmers and policymakers to test scenarios before implementing changes in the field.

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## 7 APPENDICES

## 7.1 Appendix 1: Simulation code

## ./foodweb.py

```

1 import numpy as np
2 from scipy.integrate import odeint
3 import matplotlib.pyplot as plt
4
5 # Food web interaction parameters
6 params = {
7     # Producer level
8     'growth_rate': 0.7,
9     'carrying_capacity': 1000,
10    'producer_mortality': -0.5,
11
12    # Consumer level interactions
13    'herb_consumption_rate': 0.0015,
14    'herb_conversion_efficiency': 0.15,
15    'herb_mortality': 0,
16    'sec_consumption_rate': 0.004,
17    'sec_conversion_efficiency': 0.15,
18    'sec_mortality': 0.02,
19    'tert_consumption_rate': 0.002,
20    'tert_conversion_efficiency': 0.1,
21    'tert_mortality': 0.01,
22 }
23
24 # Initial biomass for each trophic level [producers, herbivores,
25 # secondary, tertiary]
26 y0 = [600, 60, 6, 0.6]
27
28 def food_web_biomass_ode(y, t, p):
29     """Calculates biomass changes across trophic levels using
30     Lotka-Volterra equations"""
31
32     # Producer dynamics (logistic growth - herbivory - mortality)
33     dB_Pdt = (p['growth_rate'] * B_P * (1 - B_P /
34               p['carrying_capacity']) -
35               p['herb_consumption_rate'] * B_P * B_H -
36               p['producer_mortality'] * B_P)
37
38     # Consumer dynamics (consumption gains - predation losses -
39     # mortality)
40     dB_Hdt = (p['herb_conversion_efficiency'] *
41               p['herb_consumption_rate'] * B_P * B_H -
42               p['sec_consumption_rate'] * B_H * B_S -
43               p['herb_mortality'] * B_H)
44
45     dB_Sdt = (p['sec_conversion_efficiency'] *
46               p['sec_consumption_rate'] * B_H * B_S -
47               p['tert_consumption_rate'] * B_S * B_T -
48               p['sec_mortality'] * B_S)
49
50     dB_Tdt = (p['tert_conversion_efficiency'] *
51               p['tert_consumption_rate'] * B_S * B_T -
52               p['tert_mortality'] * B_T)
53
54     return [dB_Pdt, dB_Hdt, dB_Sdt, dB_Tdt]
55
56 t = np.linspace(0, 550, 5501)
57
58 solution = odeint(food_web_biomass_ode, y0, t, args=(params,))
59 B_P, B_H, B_S, B_T = solution.T

```

## ./forestplot.py

```

1 import random
2 import math
3 import matplotlib.pyplot as plt
4 import matplotlib.colors as mcolors
5 import numpy as np
6
7 rng = random.Random()
8 # piece of forest modeled as 10x10 matrix
9 land_width = 10
10 land_height = 10
11 time = 0
12 cut_forest = [[5.0] * land_width for _ in range(land_height)] #
13 # Initialize with single decimal values
14 farm_levels = [['N'] * land_width for _ in range(land_height)]
15 min_farm_squares = 16
16 max_farm_squares = 81
17 grass_growth_rate = 0.25
18 crop_growth_rate = 0.33 # yields every 3 quarters except winter
19 soil_depletion_rate = 0.01 # meaning loses 1% fertility per
20 # quarter
21
22 # set up cut forest
23 def make_farm(is_random_forest):
24     if not is_random_forest:
25         for i in range(2, land_height):
26             for j in range(1, land_width - 1):
27                 cut_forest[i][j] = 0.0 # Set to 0.0 for cut
28                 forest
29     else:
30         neighbors = set()
31         # start with a random edge spot
32         random_col = random.randint(1, land_width - 1)
33         cut_forest[land_height - 1][random_col] = 0.0 # Set to
34         0.0 for cut forest
35         neighbors.update(cell_neighbors(land_height - 1,
36                                       random_col))
37         farm_squares = random.randint(min_farm_squares,
38                                       max_farm_squares + 1)
39         for _ in range(farm_squares):
40             new_square = neighbors.pop()
41             while cut_forest[new_square[0]][new_square[1]] == 0.0
42             and len(neighbors) > 0:
43                 new_square = neighbors.pop()
44             cut_forest[new_square[0]][new_square[1]] = 0.0
45             neighbors.update(cell_neighbors(new_square[0],
46                                           new_square[1]))
47
48 # mark non-edge squares
49 for i in range(land_height):
50     for j in range(land_width):
51         neighbors = cell_neighbors(i, j)
52         is_edge = False
53         for neighbor in neighbors:
54             if cut_forest[neighbor[0]][neighbor[1]] != "N"
55             and cut_forest[neighbor[0]][neighbor[1]] != 0.0:
56                 is_edge = True
57                 break
58         if not is_edge:
59             cut_forest[i][j] = "N" # Mark as "N" if it's not
60             near the edge
61             farm_levels[i][j] = 0.0
62
63 def cell_neighbors(row, col):
64     adjacent_cells = [
65         (row - 1, col), # Up
66         (row + 1, col), # Down
67         (row, col - 1), # Left
68         (row, col + 1) # Right
69     ]
70
71     filtered_neighbors = []
72     for row_index, col_index in adjacent_cells:
73         # Check if the cell is within bounds
74         if 0 <= row_index < land_height and 0 <= col_index <
75         land_width:
76             filtered_neighbors.append((row_index, col_index))
77
78     return filtered_neighbors
79
80 # randomize forest
81 def randomize_forest():
82     for i in range(land_height):
83         for j in range(land_width):
84             if cut_forest[i][j] != "N" and cut_forest[i][j] !=
85             0.0:
86                 cut_forest[i][j] = round(rng.uniform(3, 5), 1) #
87                 Randomize values between 3 and 5
88
89 def sim_edge_growth(curr_forest):
90     new_forest = [row[:] for row in curr_forest] # Deep copy of
91     list
92     for i in range(land_height):
93         for j in range(land_width):
94             if cut_forest[i][j] != "N":
95                 rel_regrow_rate =
96                 cell_regrowth_total(curr_forest, i, j)

```

```

83         new_forest[i][j] =
            forest_level_inc(cut_forest[i][j],
84         return new_forest
85
86 def cell_regrowth_total(curr_forest, row_index, col_index):
87     total_regrowth = 0.0
88     neighbors = cell_neighbors(row_index, col_index)
89     for row, col in neighbors:
90         if curr_forest[row][col] == "N":
91             pass
92         else:
93             total_regrowth += curr_forest[row][col]
94
95     return total_regrowth
96
97 def forest_level_inc(curr_level, regrowth_rate):
98     growth = curr_level + random.random() * ((grass_growth_rate *
99     return min(5.0, round(growth, 2)) # Update level with
100     regrowth
101
102 def sim_crop_growth(curr_farm):
103     new_farm_levels = [row[:] for row in curr_farm] # Deep copy
104     of farm levels
105     num_years = int(time / 4)
106
107     for i in range(land_height):
108         for j in range(land_width):
109             if curr_farm[i][j] != "N": # Only update farmed land
110                 if time % 4 == 0: # Every 4 time steps, reset
111                     crop level
112                     new_farm_levels[i][j] = 0.5
113                 else:
114                     # Crop growth follows a logistic growth curve
115                     with nutrient depletion effects (assumes
116                     first year of crop growth has perfect soil)
117                     growth = curr_farm[i][j] + ((crop_growth_rate
118                     * curr_farm[i][j] * (1 - curr_farm[i][j] / (5
119                     * math.exp(-1 * soil_depletion_rate *
120                     time))))
121                     new_farm_levels[i][j] = min(5.0,
122                     round(growth, 2))
123
124     return new_farm_levels
125
126 def print_farm():
127     print("Forest")
128     for row in cut_forest:
129         print(row)
130     print("\nFarm")
131     for row in farm_levels:
132         print(row)
133
134 make_farm(True)
135 randomize_forest()
136 # Test for 1 year essentially
137
138 for _ in range(4):
139     cut_forest = sim_edge_growth(cut_forest)
140     farm_levels = sim_crop_growth(farm_levels)
141     time += 1
142     print_farm()
143
144 forest_cmap = plt.get_cmap('Greens')
145 norm = mcolors.Normalize(vmin=0, vmax=5)
146
147 farm_color = '#D2B48C'
148
149 neutral_color = '#F0F0F0'
150
151 color_grid = np.zeros((land_height, land_width, 3))
152
153 for i in range(land_height):
154     for j in range(land_width):
155         if farm_levels[i][j] != "N":
156             color_grid[i, j] = mcolors.to_rgb(farm_color)
157         elif cut_forest[i][j] == "N":
158             color_grid[i, j] = mcolors.to_rgb(neutral_color)
159         elif cut_forest[i][j] == 0.0:
160             color_grid[i, j] = (0.0, 0.0, 0.0)
161         else:
162             normalized_val = cut_forest[i][j]
163             color = forest_cmap(norm(normalized_val))[:3]
164             color_grid[i, j] = color
165
166 fig, ax = plt.subplots(figsize=(6,6))
167 ax.imshow(color_grid, extent=[0, land_width, 0, land_height])
168
169 ax.set_xticks(np.arange(0, land_width+1, 1))
170 ax.set_yticks(np.arange(0, land_height+1, 1))
171 ax.grid(color='black', linewidth=1)
172 ax.set_xticklabels([])
173 ax.set_yticklabels([])
174
175 import matplotlib.patches as mpatches
176 from matplotlib.colors import ListedColormap, BoundaryNorm
177
178 farm_patch = mpatches.Patch(color=farm_color, label='Farm')
179
180 forest_light = forest_cmap(norm(1.25))
181 forest_dark = forest_cmap(norm(4.0))
182 forest_light_patch = mpatches.Patch(color=forest_light[:3],
183 label='Light Forest')
184 forest_dark_patch = mpatches.Patch(color=forest_dark[:3],
185 label='Dark Forest')
186
187 plt.legend(handles=[farm_patch, forest_light_patch,
188 forest_dark_patch],
189           bbox_to_anchor=(1.05, 1), loc='upper left')
190
191 plt.title('Forest and Farm Choropleth')
192
193 plt.show()

```

## ./graph.py

```

1 import numpy as np
2 from scipy.optimize import linprog
3
4 def optimize_food_web(S, E, L, U, eta, edges):
5     num_edges = len(edges)
6     c = -np.ones(num_edges) # Objective: maximize total energy
7     transfer
8     A_eq = [] # Equality constraint matrix
9     b_eq = [] # Equality constraint vector
10    # Map edges to index
11    edge_idx = {edge: i for i, edge in enumerate(edges)}
12
13    # Energy balance constraints
14    for i in range(S):
15        row = np.zeros(num_edges)
16        for j in range(S):
17            row[edge_idx[(i, j)]] = 1 #
18            Outflow
19            if (j, i) in edge_idx:
20                row[edge_idx[(j, i)]] =
21                -eta # Inflow
22            A_eq.append(row)
23            b_eq.append(L[i] - E[i])
24
25    # Maximum energy capacity constraints
26    A_ub = np.zeros((S, num_edges))
27
28    b_ub = np.array(U)
29    for i in range(S):
30        for j in range(S):
31            if (i, j) in edge_idx:
32                A_ub[i, edge_idx[(i, j)]] = 1
33
34    # Non-negativity constraints
35    bounds = [(0, None) for _ in range(num_edges)]
36
37    # Solve linear program
38    res = linprog(c, A_ub=A_ub, b_ub=b_ub, A_eq=A_eq,
39                 b_eq=b_eq, bounds=bounds, method='highs')
40
41    if res.success:
42        return {edges[i]: res.x[i] for i in range(num_edges)}
43
44 S = 3 # Number of species
45 E = [10, 5, 0] # Energy production per species
46 L = [2, 3, 4] # Energy loss per species
47 U = [15, 10, 8] # Maximum energy intake per species
48 eta = 0.1 # Trophic transfer efficiency
49 edges = [(0, 1), (1, 2)] # Food web structure
50
51 optimal_flows = optimize_food_web(S, E, L, U, eta, edges)

```

## 7.2 Appendix 2: One-page letter to farmer

To whom this may concern,

Thank you for expressing your interest in considering a shift to organic farming practices! As you navigate this complicated decision process, we'd like to offer you some insights from our recent research to help you make a more informed decision. Our paper focused mainly on the impacts of different agricultural practices on plant health and species populations in the surrounding ecosystem. We concluded that while organic farming is certainly more friendly to the environment, you should be aware of possible challenges that organic farming practices could bring about.

One of the key findings of our research is that without the use of herbicides and pesticides, crops become more vulnerable to diseases and pests, respectively. From simulating species populations in various agricultural environments, we found that pest populations fluctuated most dramatically in organic systems. This increased volatility increases the risk of pest outbreaks, threatening crop yields and creating constant instability in the ecosystem. We also noted that in the absence of herbicides and pesticides, the seasonal cycles of producers (like your crops) and herbivores (pests) became less regulated. This lack of regulation can result in significant variations in crop health over time, making it more difficult to predict and manage your harvests effectively.

From an economic standpoint, these challenges can increase costs and risks. Specifically, the threat of reduced yields demands an alternative form of pest control and can hamper your farm's profitability. While organic produce tends to have higher market prices, it is important to consider the potential profit and environmental benefits against the risks of crop losses and the expenses associated with more sustainable practices.

In contrast, conventional farming practices that utilize herbicides and pesticides can help maintain more stable populations of both crops and pests. Our ecosystem models reinforced this conclusion, suggesting that using these chemicals can lead to more consistent and predictable yields, which plays an important role in a farm's success.

Of course, ecosystem stability and protection provide long-term impacts, which incentivize integrated approaches that balance these concerns with economic viability. Adopting practices like Integrated Pest Management (IPM) can reduce reliance on chemicals while effectively controlling pests through a combination of biological controls, crop rotation, and targeted chemical use. At the same time, collaborating with organizations such as the National Sustainable Agriculture Coalition (NSAC) and the Organic Farmers Association (OFA) or advocating for the Organic Opportunities Act will help organic farmers receive more benefits as compensation for higher risk levels or costs.

Overall, considering organic farming is a constant process of weighing potential environmental benefits against the practical challenges and economic implications. We hope that our insights and recommendations will help you make an informed choice that aligns with your values while accommodating the fluctuations inherent in agriculture. If you'd like to further discuss our findings, please don't hesitate to reach out.

Wishing you a bountiful harvest and a flourishing ecosystem!

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